Exam 1 **Math 314** **Fall 2006**

Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

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(24) **1.** Consider the linear system given by the following:

$$\begin{aligned} x_1 + x_2 + x_3 - x_4 &= 2 \\ 2x_1 + x_2 - 2x_4 &= 1 \\ 2x_1 + 2x_2 + 2x_3 - 2x_4 &= 4 \end{aligned}$$

(a) (12) Use Gauss-Jordan elimination to find the general solution to this system. Clearly specify the elementary row operations you use.

(b) (4) If we write the system as $A\mathbf{x} = \mathbf{b}$, what are the coefficient matrix A and right-hand-side vector $\mathbf{b}$? What are the rank and nullity of A?

(c) (3) Express the reduced row echelon form R of the augmented matrix $\widetilde{A}$ of this system as product of elementary matrices and $\widetilde{A}$.

(d) (5) Apply the row operations used in part (a) in the same order as in (a) to a general right hand side vector $\mathbf{b} = (b_1, b_2, b_3)$. What is the resulting vector?

(16) **2.** Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$. Find the inverse of A and use it to solve $A\mathbf{x} = \mathbf{b}$ with $\mathbf{b} = (2, -4, 8)$.

(12) **3.** Solve the following systems for the (complex) variable z. Express your answere in standard form ($z = x + \mathrm{i}y$) where possible.

(a) $z = \mathrm{e}^{\mathrm{i}\pi} + 2\mathrm{i}$.

(b) $(2 + \mathrm{i})\, z = 1$

(c) $z^3 = 1$

(20) **4.** Carry out these calculations or indicate they are impossible. You are given that $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1+\mathrm{i} \\ 0 & 1 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$.

(a) $\mathbf{y}C\mathbf{x}$

(b) $\mathbf{x}\mathbf{y}$

(c) $\mathbf{x} + 2\mathbf{x}^T$

(d) D^*

(e) C^{-1}

(f) CD

(18) **5.** Fill in the blanks or answer True/False. Parts (a) and (b) are worth 3 points and remaining parts are worth 2 points.
(a) If A is a 2×2 nonzero matrix and the system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for some $\mathbf{b}$ then rank A =____________ and A ____________ invertible. (Fill in "is" or "is not".)

(b) $T((x,y)) = (x+y, 2x, 4y-x)$ is a matrix multiplication function $T_A((x,y))$, where $A =$

(c) If a Markov chain has transition matrix $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$, and initial state $x^{(0)} = (1,0)$, then $x^{(2)} =$

(d) As a matrix–vector product, $x_1 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} =$

(e) If 3×3 matrix A is invertible, then the reduced row echelon form of A is:

(f) Any homogeneous (right-hand-side vector $\mathbf{0}$) linear system is consistent (T/F):

(g) If A, B are 2×2 matrices, then $(AB)^2 = A^2B^2$ (T/F):

(h) Every diagonal matrix is symmetric (T/F):

(10) **6.** Let $A = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 4 & 1 & 3 \end{bmatrix}$.
(a) Verify the commutative law of matrix addition for these two matrices.

(c) (Honors only) Give a proof of this law.