Quiz 5 Key **Math 221H** **Fall 2008**

Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin\pi$. Point values of problems are given in parentheses. Notes or text in *any* form are not allowed. The only electronic equipment allowed is a calculator.

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(7) **1.** (Exer. 5.4.x) Use the eigenvalue method to find a general solution to the linear system $\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{x}$.

SOLUTION. The eigenvalues satisfy $0 = (3-\lambda)(1-\lambda) - -1 = \lambda^2 - 4\lambda + 4 = (\lambda-2)^2$, so $\lambda = 2, 2$. An eigenvector $\mathbf{v}_1 = (a, b)$ satisfies $0 = (3-\lambda)a - 1b = a - b$ and $0 = 1a + (1-\lambda)b = a - b$, that is, $a = b$. Take $a = 1$ and get eigenvector $\mathbf{v}_1 = (1, 1)$.

For a generalized eigenvector $\mathbf{v}_2 = (a, b)$, solve $1 = (3-\lambda)a - 1b = a - b$ and $1 = 1a + (1-\lambda)b = a - b$, so take $b = 1$, get $a = 2$ and so $\mathbf{v}_2 = (2, 1)$.

Thus a general solution is

$$\mathbf{x}(t) = c_1\mathbf{v}_1 e^{2t} + c_2(\mathbf{v}_1 t + \mathbf{v}_2)e^{2t} = \left(c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} t+2 \\ t+1 \end{bmatrix}\right) e^{2t}$$

(6) **2.** (Exer. 6.1.23) Solve the equation $\frac{dy}{dx} = \frac{G(x,y)}{F(x,y)}$ to find trajectories of the system $dx/dt = y = F(x, y)$, $dy/dt = -x = G(x, y)$ and sketch a few.

SOLUTION. The equation becomes $\frac{dy}{dx} = \frac{-x}{y}$ so that $ydy = xdx$. Integrate and obtain

$$\frac{x^2}{2} = -\frac{y^2}{2} + C.$$

Multiply by 2 (absorbed by C) and collect terms to obtain $x^2 + y^2 = C = r^2$, which gives concentric circles with center at the origin, which is the unique critical point. Draw a picture of these.

(7) **3.** (Exer. 7.2.1) Use Laplace transforms to solve the IVP $x'' + 4x = 0$, $x(0) = 5$, $x'(0) = 0$.

SOLUTION. Apply the Laplacian with $X = \mathcal{L}\{x\}$ and obtain from linearity and derivatives (Table #1 and #2) that

$$\mathcal{L}\{x'' + 4x\} = \mathcal{L}\{x''\} + 4\mathcal{L}\{x\} = s^2X - sx(0) - x'(0) + 4X = s^2X - 5s + 4X = 0,$$

so that

$$X(s) = \frac{5s}{s^2+4} = 5\frac{s}{s^2+2^2} = \mathcal{L}\{5\cos 2x\}$$

and therefore $x(t) = \mathcal{L}^{-1}\{X(s)\} = 5\cos 2x$.