Sample Exam 1 **Math 221H** **Fall 2008**

Name:_____ **Score:**_____

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin \pi$. Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. No electronic equipment is allowed.

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(16) **1.** Specify the order of the following ODEs and classify (do *not* solve) the first order ODEs as autonomous(A), exact(E), linear(L) or separable(S), if any. Also indicate what method you would use to solve them (do not actually do it.)

(a) $y\,y' = x\left(y^2+1\right)$

(b) $yy'' + (y')^2 = 0$

(c) $\dfrac{dy}{dx} - \dfrac{y}{2x} = 2xy^{-1}$

(d) $\left(3x^2+2y^2\right) + \left(4xy+6y^2\right)\dfrac{dy}{dx} = 0$

(14) **2.** A tank contains 1000 liters (L) of a solution of 100 kg of salt dissolved in water. Pure water is pumped in at a rate of 5 L/s and the stirred solution is pumped out at the same rate. Assume that $S(0) = 100$, find a differential equation condition for the amount $S(t)$ of salt in the tank at time t and solve for S.

(27) **3.** Find all solutions to these DEs or IVPs:

(a) $\dfrac{dy}{dx} = y\sin x$

(b) $(4x-y) + (6y-x)\dfrac{dy}{dx} = 0$

(c) $y' + 3y = 2xe^{-3x},\ \ y(0) = 2$

(12) **4.** The acceleration of a Maserati is proportional to the difference between 250 km/hr and the velocity of this sports car. Set up a DE for velocity of this car, sketch its phase diagram and sketch a few representative solutions to the DE.

(16) **5.** You are given the DE $\dfrac{dy}{dx} = \sqrt[3]{y}$.

(a) Find the equilibrium solutions to this DE and classify them as stable or unstable.

(b) Determine the extent to which the existence/uniqueness theorem applies to the given DE with initial condition $y(0) = 0$.

(15) **6.** A population is modeled by the autonomous differential equation $\dfrac{dx}{dt} = 4x - x^2 - h$ with parameter h. Sketch the bifurcation diagram for this DE and label the critical points as stable or unstable. Sketch some typical solutions to the DE for $h = 2$.