Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin\pi$. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator. Table entries for Laplace transforms may be used freely, unless otherwise stated. However, you must show enough detail so that it is clear that you are using table entries.

---

(10) **1**. Classify each of the following systems as linear(L), nonlinear (NL), autonomous(A) or non-autonomous (NA).

(a) $3y^2y' + y^3 = e^x$

(b) $\dfrac{dy}{dt} + \sin(y) = 0$

(c) $\dfrac{dx}{dt} = y, \dfrac{dy}{dt} = x + y$

(d) $\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} = t^2y$

(14) **2**. Solve these DEs and IVPs

(a) $\left(1+x^2\right)\dfrac{dy}{dx} = 2y$

(b) $\dfrac{1}{y}\dfrac{dy}{dx} = y^3(2x+1), \qquad y(0) = 1.$

(c) $\dfrac{dy}{dt} - \dfrac{2}{t}y = t^2e^t.$

(12) **3**. Use Euler's Method with a stepsize of $h = 0.5$ to compute the approximate solution on the interval [0,1] to the IVP

$\dfrac{dy}{dx} = 1 + xy^2, \qquad y(0) = 1.$

(10) **4**. Sketch the phase line, classify equilibria and sketch representative solutions for the equation $dy/dx = y(y^2-1)$.

(12) **5**. Find the general solution to this damped oscillator problem: $2y'' + 4y' + 2y = 0$.

(14) **6**. Compute the equilibria and trajectories of the system $dx/dt = (1-y)$, $dy/dt = xy$.

(15) **7**. Given that the linear system $Y' = AY$ with constant coefficient matrix A has eigenvalues -1, -2, with corresponding eigenvectors $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, respectively, write out the straight line solutions and the general solution to the system. Use this to find the solution to the system that satisfies $Y(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

(15) **8**. Use the eigenvalue method to find the general solution to the linear system $x' = x + y, \quad y' = -x + y$. (You

(6) **9**. A linear system $Y' = AY$ has repeated eigenvectors $2, 2$, eigenvector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and generalized eigenvector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Use these facts to find a general solution to the system and write out the form of each component of this solution.

(24) **10**. Given the IVP $y'' + 3y' + 2y = t$, $y(0) = 0$, and $y'(0) = 0$.
(a) Find the general solution to the differential equation by the method of undetermined coefficients.

(b) Find the solution to the IVP by using Laplace transforms.

(14) **11**. In each case below, find the *form* of a particular solution to the differential equation. (Do NOT explicitly determine the unknown coefficients.)
(a) $(D(D-2)(D-1))[y] = t + e^{3t}$

(b) $y'' + 5y' + 6y = \cos(4t)$

(14) **12**. Find general solutions to the following equations
(a) $y'' - 4y' + 4y = 0$

(b) $y''' + y' = 0$

(12) **13**. You are given that the function $f(t)$ is defined by $f(t) = \begin{cases} 0, & \text{if } t < 1, \\ t, & \text{if } 1 \le t < 4, \\ 0, & \text{if } 4 \le t. \end{cases}$
(a) Compute $\mathcal{L}\{f(t)\}$ directly from the definition of Laplace transform.

(b) Express $f(t)$ in terms of step functions and use the tables to compute $\mathcal{L}\{f(t)\}$.

(18) **14**. Find the inverse Laplace transforms.
(a) $Y(s) = \dfrac{1}{s(s^2+4)} + \dfrac{1}{s^4}$, $\mathcal{L}^{-1}\{Y(s)\} =$

(b) $Y(s) = \dfrac{s+3}{(s-1)^2+4}$, $\mathcal{L}^{-1}\{Y(s)\} =$

(c) $Y(s) = \dfrac{1}{(s-1)(s+1)(s+2)}$, $\mathcal{L}^{-1}\{Y(s)\} =$

(10) **15**. Find the Laplace transforms.
(a) . $\mathcal{L}\left\{e^{-3t}\cos 2t\right\} =$

(b) $\mathcal{L}\left\{\int_0^t e^{3v}(t-v)\,dv\right\} =$