Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin\pi$. Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

(Note: This sample exam is about 1.5 questions too long.)

(18) **1.** Solve the IVP $\dfrac{dx_1}{dt} = -2x_1 + x_2$, $\dfrac{dx_2}{dt} = 4x_1 - 2x_2$, $x_1(0) = 1$, $x_2(0) = 0$ by the method of eigenvalues. (Arithmetic check: eigenvalues are $0, -4$.)

(18) **2.** Find the general solution to $\dfrac{dx_1}{dt} = -2x_1 + x_2$, $\dfrac{dx_2}{dt} = -1x_1 - 4x_2$, by the method of eigenvalues. (You may assume that $-3, -3$ are the eigenvalues of this system and $[1, -1]^T$ is an eigenvector.)

(10) **3.** Sketch typical phase planes for a linear system $dx/dt = Ax + By$, $dy/dt = Cx + Dy$ which has at the origin a

(a) stable node. (b) saddle. (c) center.

(10) **4.** Solve the DE $\dfrac{dx}{dt} = 2y$, $\dfrac{dy}{dt} = -4x$, sketch a few representative solutions and classify the critical point at the origin.

(12) **5.** Find the inverse Laplace transforms.

(a) $Y(s) = \dfrac{1}{(s+9)\,s^2}$. $\mathcal{L}^{-1}\{Y\} =$

(b) $Y(s) = \dfrac{s}{s^2 - 2s + 5} + \dfrac{1}{s^4}$. $\mathcal{L}^{-1}\{Y\} =$

(12) **6.** Find the Laplace transforms.

(a) $\mathcal{L}\left\{\dfrac{\sinh t}{t}\right\} =$

(b) $\mathcal{L}\left\{e^{3t}t^2 + t^3\cos(2t)\right\} =$

(12) **7.** Express $f(t) = \int_0^t e^v \sin(t - v)dv$ in terms of convolutions. Then use this expression to find the Laplace transform of $f(t)$.

(18) **8.** Solve the IVP

$$y'' - 4y = e^{2t},\ y(0) = 0,\ y'(0) = 0$$

by the method of Laplace transforms.