Sample Exam 2 **Math 208** **Spring 2008**

Name:_____ **Score:**_____

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin \pi$. Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

(Note: This sample exam is about one question too long.)

(15) **1.** Given that $y_1(x) = \ln(x)$ solves the ODE $xy'' + y' = 0$, use reduction of order to find a second solution $y_2(x)$ and show that $y_1(x)$, $y_2(x)$ form a fundamental set by calculating the Wronskian of these functions.

(10) **2.** Convert this damped oscillator problem to a first order linear system and write it in vector form: $y'' + 2y' + 4y = 0$. Exhibit the solution to this first order system.

(15) **3.** Find a general solution to the DE $y'' - 2y' + 3y = x + 1$.

(15) **4.** Consider the differential equation $L[y] = x^2y'' - 2y = x^3$.

(a) Express the operator L in terms of the derivative operator D. Is L a linear operator?

(b) Show that $y_1(x) = 1/x$ and $y_2(x) = x^2$ are linearly independent solutions to the associated homogeneous equation.

(c) Add initial conditions $y(1) = 0$, $y'(1) = 1$ to the differential equation and solve the resulting initial value problem.

(15) **5.** Consider the ordinary differential equation (ode) $y'' - xy' + y = 0$.

(a) Verify that $f(x) = x$ is a solution to this ode.

(b) Use reduction of order to obtain a second solution in integral form and verify that this solution and $f(x)$ form a linearly independent set of solutions.

(15) **6.** Derive the general solution to each of the following DEs or IVPs

(a) $y''' - 4y'' = 0$.

(b) $y'' + 2y' + 3y = 0$.

(c) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 0$.

(15) **7.** Set up the form for a particular solution to the following differential equations using the method of undetermined coefficients. Only solve for the coefficients in part (a).

(a) $y'' - y = \sin x$.

(b) $y^{(4)} + 9y'' = x \sin 3x$.