Sample Exam 1 **Math 208** **Spring 2008**

Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin \pi$. Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

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(12) **1.** Classify the following ODEs as separable(S), linear(L) or autonomous(A). Indicate what method you would use to solve it (but do not do so.)
(a) $dy/dt = (ty)^2$
(b) $t(dy/dt) + y = 0$
(c) $dy/dx = 1 + y^2$
(d) $d^2y/dx^2 = 1 + (y')^2$

(24) **2.** Find all solutions to these DEs or IVPs.
(a) $\dfrac{dx}{dt} = (tx)^2$
(b) $\dfrac{dy}{dx} = \dfrac{x+3y}{y-3x}$
(c) $t\dfrac{dy}{dt} = 2y + t^3, \qquad y(1) = 0$

(16) **3.** Find the phase line and classify the equilibrium points for the autonomous differential equation $\dfrac{dw}{dt} = (w-1)(w-2)^2$. Use this to roughly sketch typical solutions $w(t)$.

(16) **4.** A population of elephants is currently 20,000. Elephant growth rate under ideal conditions is about 1.1 per year. However, the carrying capacity of the environment is only 80,000 elephants. Set up a differential equation that models this data and use its phase line to sketch (roughly) the solution.

(16) **5.**. A 100 gallon tank is half full at time $t = 0$. At $t = 0$ we begin dumping 2 gallons per minute of salt solution containing 0.25 pounds of salt per gallon into the tank. At the same time we begin removing 1 gallon of (fully mixed) salt water from the tank per minute. Set up an initial value problem for the amount of salt in the tank. For what time is this model valid?

(16) **6.** Verify that $y_1(t) = 0$ and $y_2(t) = e^{-1/t}$ are solutions to the differential equation $\dfrac{dy}{dt} = \dfrac{y}{t^2}$. Now let $y(t)$ be a solution for $t \geq 1$ such that $y(1) = 1/4$. Use the Existence/Uniqueness Theorem to show there is a solution $y(t)$ and $0 \leq y(t) \leq 1$. (Hint: $e^{-1/t} \leq 1$.)