Name: ______________________ **Lecturer:** ______________________

Instructions: This exam should have 8 pages; check that it does. Show all your work for full credit. You may use the back of exam pages for scratch work, but clearly indicate if you want this work graded. Calculators are the only electronic device allowed (no cell phones, pocket pcs, etc.), but *an answer will only be counted if it is supported by all the work necessary to get that answer.* Make obvious simplifications: for example, write $\sqrt{2}/2$ instead of $\cos(\pi/4)$ for an answer. Also, give exact answers only, except as noted; for example, write π instead of 3.1415 if π is the answer. Notes or text *in any form* are not allowed. Unless otherwise stated, parts of problems have approximately equal value.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total
Points	18	6	12	12	16	14	16	14	14	12	15	10	12	15	14	200
Score																

1. (18 points) (a) Let $\mathbf{a} = \langle 2, 0, 1\rangle$ and $\mathbf{b} = \langle 1, 1, 0\rangle$. Compute $\text{comp}_{\mathbf{b}}\mathbf{a}$

(b) Find an equation of the plane containing $(0, 0, 0)$ and parallel to vectors $\mathbf{a}$ and $\mathbf{b}$ from (a).

(c) Find parametric equations for the line passing through the point $(4, -2, 0)$ and parallel to vector $\mathbf{a}$ from (a).

2. (6 points) At what points is the function $f(x, y) = \dfrac{2x - y^2}{xy - 4x}$ continuous?

3. (12 points) Let $f(x, y, z) = xy^2z$.

(a) Find a vector in the direction in which $f(x, y, z)$ increases most rapidly, starting at the point $(1, 2, 3)$, and the value of the maximum rate of change at that point.

(b) Find the rate of change of $f(x, y, z)$ at $(1, 2, 3)$ in the direction from this point toward the point $(3, 0, 2)$.

4. (12 points) Find the differential of $z = f(x, y) = 2 + x^2 \cos(xy)$ and the tangent plane to the graph of this function at $(x, y) = (1, \pi/2)$.

5. (16 points) Find the absolute extrema of the function $f(x, y) = x + 2y$ on the disk $x^2 + y^2 \leq 5$.

6. (14 points) An object moves along a trajectory C with position vector $\mathbf{r}(t)$ and acceleration vector $\mathbf{a}(t) = \left\langle e^t, t, \cos t \right\rangle$. It is given that $\mathbf{v}(0) = \langle 4, -2, 4 \rangle$ and $\mathbf{r}(0) = \langle 0, 4, -2 \rangle$. Find the position and velocity vectors as functions of time t.

7. (16 points) Let $f(x, y) = x^2 y - x^2 - 2y^2$. Find the critical points of $f(x, y)$ and classify them as local maxima, minima or saddle points.

8. (14 points) By reversing the order of integration, evaluate the iterated integral $\int_0^3 \int_{y^2}^9 y \sin(x^2)\, dx\, dy$.

9. (14 points) Consider the integral $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \left(x^2+y^2+z^2\right) dz\, dx\, dy$.

Apply a change of coordinates from rectangular to spherical and evaluate the resulting iterated integral.

10. (12 points) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = \langle y^2 e^{2x} - y + 2x, ye^{2x} - x - 1 \rangle$ and C is an unknown path from $(2, 3)$ to $(3, 0)$.

11. (15 points) Use Green's Theorem to evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle y^2 + 3x^2 y, xy + x^3 \rangle$ and C is formed by $y = x^2$ and $y = 2x$ oriented positively with respect to the region bounded by these curves.

12. (10 points) If $z = f(u, v) = u^v$, $u = 2x^2 + 4$ and $v = 7x^3 + 1$, use the chain rule to find $\frac{dz}{dx}$. Leave your answer in terms of u, v and x.

13. (12 points) Find $\iint_R y\,dA$, where R is the region above the line $y = -x$ and inside the circle $x^2 + y^2 = 2x$, by setting up and evaluating an integral in polar coordinates.

14. (15 points) Set up and evaluate $\iint_S (2x+z)\,dS$, where S is the portion of $2x+4y+z=12$ that is bound by the three coordinate planes (i.e., in the first octant.)

15. (14 points) Let $\mathbf{F}(x,y,z) = (2xy+3x-1)\,\mathbf{i} + \left(4y-3y^2+e^z\right)\mathbf{j} + \left(xy^5-2z+4yz\right)\mathbf{k}$ and use the Divergence Theorem to find the flux of $\mathbf{F}$ over ∂Q, where Q is the solid $x^2+y^2+z^2 \le 4$.