Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin \pi$. Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. The only electronic equipment allowed is a calculator.

(24) **1.** Let $f(x, y) = 8x^2 + 4x^2y + y^2 - 7$.

(a) Find all derivatives up to the second order.

(b) Find all critical points of f.

(c) Use the second derivative test to classify the critical points of f.

(22) **2.** Let $f(x, y) = xy$.

(a) Find the extrema of f subject to the constraint $x^2 + 2y^2 = 1$ by the method of Lagrange multipliers.

(b) What additional point(s) should you check to find the absolute extrema of f over the region $x^2 + 2y^2 \leq 1$?

(18) **3.** Express the volume of the solid bounded above by the paraboloid $z = x^2 + y^2$ and below by the rectangle $R : 0 \leq x \leq 1, 0 \leq y \leq 1$ as a double integral and evaluate this integral.

(18) **4.** Evaluate the integral

$$\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,dy\,dx$$

by interchanging the order of integration. Clearly sketch the region of integration.

(18) **5.** Convert the iterated integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right) dy\,dx$ to polar coordinates and evaluate. Sketch the region of integration for this problem. What is the average value of $f(x,y) = x^2 + y^2$ over this region?