

Standard curves in polar coordinates

First, be aware that replacing θ everywhere by $\theta - \theta_0$ in any of these equations simply rotates the graph counterclockwise around the origin by θ_0 . Thus, for example, since $r = 2 \cos(\theta)$ is a circle passing through the origin with center on the x -axis, $r = 2 \cos(\theta - \frac{3\pi}{4})$ is a circle going through the origin with center in the 2nd quadrant (on the line $y = -x$). Finally, replacing (r, θ) by $(-r, \theta + \pi)$ will change an equation where r is positive at a point to one where r is negative at that point, or vice versa. This is useful since r must be positive in our polar double integrals.

In the following table, a , b , and c are all constants with the indicated restrictions.

Equation	Shape
$r = c, c > 0$	Circle of radius c centered at the origin.
$\theta = c$	Ray (half-line) starting at the origin (full line if you allow $r < 0$).
$r = a \cos(\theta) + b \sin(\theta)$, a and b not both 0	Circle through the origin with center at $(\frac{a}{2}, \frac{b}{2})$. In rectangular coordinates, this is $x^2 + y^2 = ax + by$.
$r = a \cos(n\theta)$ or $r = a \sin(n\theta), a \neq 0$	If $n > 1$ is an odd integer, these are n -leaf clovers. If $n > 1$ is an even integer, these are $2n$ -leaf clovers.
$r = a + b \cos(\theta)$ or $r = a + b \sin(\theta), ab \neq 0$	Cardioid - classic heart shape with the cusp at the origin if a and b have the same absolute value.
$r^2 = a \cos(2\theta)$ or $r^2 = a \sin(2\theta), a \neq 0$	Lemniscate (shape of ∞) with center at origin. With the cosine and $a > 0$, runs horizontal, with cosine and $a < 0$, runs vertical. With the sine, runs diagonally along $y = x$ or $y = -x$ depending on the sign of a .
$r = a\theta + b, a \neq 0$	Spiral, if $a > 0$ goes outward as you move counterclockwise, if $a < 0$ goes outward as you move clockwise.