Standard surfaces in cylindrical coordinates

As in polar coordinates, be aware that replacing θ everywhere by $\theta-\theta_0$ in any of these equations simply rotates the surface counterclockwise (as viewed from above) around the z-axis by θ_0. And again, replacing (r,θ,z) by $(-r,\theta+\pi,z)$ will change an equation where r is positive at a point to one where r is negative at that point, or vice versa. This is useful since r must be positive in our cylindrical triple integrals, as in our polar double integrals.

In the following table, c is a constant with the indicated restrictions.

Equation	Shape
$r=c$, $c>0$	Right circular cylinder of radius c centered around the z-axis.
$\theta=c$	Half-plane starting at the z-axis (full plane if you allow $r<0$).
Any equation involving only r and θ, with no z	Look at the equation as a polar graph in the *xy*-plane, then translate that parallel to the z-axis to get a "cylinder" with that cross-section. For example, $r^2=2\cos(2\theta)$ gives a cylinder with a figure 8 (or ∞) cross-section with the center running up and down the z-axis. Likewise, $r=3\cos(5\theta)$ gives a cylinder whose cross-section is a 5-leaf clover.
Any equation involving only z and r, not involving θ	Graph this in the rz-plane (or half-plane if you already have made sure $r\ge 0$), and the surface is what you get by rotating that around the z-axis. Thus e.g. $z=2\sqrt{x^2+y^2} \Leftrightarrow z=2r$ with $r\ge 0$ is a ray going upward from the origin with slope 2 in the rz-half-plane, becomes a cone going upward with its vertex at the origin when you rotate that around the z-axis. $z=r^2$ is a parabola in the rz-plane, rotates to give a circular paraboloid centered around the z-axis. $(r-2)^2+z^2=1$ is a circle in the right half of the rz-plane, which rotates to give a torus (donut shape)!

Almost all other surfaces used in cylindrical integrals can be made to fit the format $z=f(r,\theta)$, and lead to integrals where you want to integrate z first, and project (flatten) the volume onto the *xy*-plane where the shape and integral become good in polar coordinates. ($r=f(z,\theta)$ is possible but rare.) E.g., if we want $\iiint\limits_R (x^2+y^2)\,dV$ where R is bounded by $z=x^2+y^2$ and $z=4x$, in cylindricals those sides are $z=r^2$ and $z=4r\cos(\theta)$, and the integral we want is $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{4\cos(\theta)}\int_{r^2}^{4r\cos(\theta)} r^3 dz\ dr\ d\theta$.

Note that in the *xy*-plane, the volume projects to the inside of the circle $x^2+y^2=4x \Leftrightarrow r=4\cos(\theta)$.