Name: ______________________ **Lecturer:** ______________________

Instructions: This exam should have 7 pages; please check that it does. Show all your work for full credit. Calculators are allowed, but *an answer will only be counted if it is supported by all the work necessary to get that answer.* Simplify as much as possible, except as noted: for example, write $\sqrt{2}/2$ instead of $\cos(\pi/4)$ for an answer. Also, give exact answers only, except as noted; for example, write π instead of 3.1415 if π is the answer. Notes or text *in any form* are not allowed.

Problem	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
Value	12	12	12	12	14	18	16	16	14	14	16	14	14	16	200
Score															

1. (12 points) A force $\mathbf{F} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ is applied at a point on the plane given by $2x - y + 2z = 1$.

(a) Find a vector $\mathbf{a}$ that is normal to the plane.

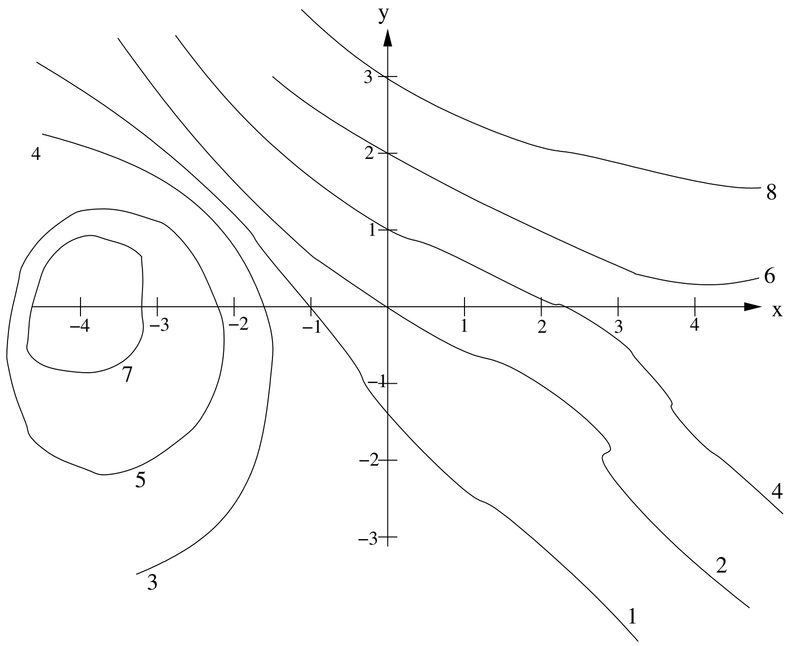
(b) Find the $\operatorname{proj}_{\mathbf{a}} \mathbf{F}$, the projection of F along $\mathbf{a}$, and show that it is orthogonal to $\operatorname{orth}_{\mathbf{a}} \mathbf{F} = \mathbf{F} - \operatorname{proj}_{\mathbf{a}} \mathbf{F}$.

2. (12 points) An object travels with position vector $\mathbf{r}(t) = \langle t, e^{-t}, \cos 2t \rangle$, where t is time.

(a) Find the velocity and acceleration of object.

(b) At what positive points in time (if any) is the velocity of the object parallel to the xy-plane?

3. (12 points) A function $z = f(x, y)$ has the following contour graph. Use this graph to estimate $\bigtriangledown f(0,0)$. Are there any points in this graph that are good candidates for local maxima of $f(x, y)$?



4. (12 points) Let Q be a solid inside the paraboloid $z = x^2 + y^2$ and below the plane $z = 4$ with constant mass density ρ. Sketch this solid. Express the mass and the first coordinate of the center of mass of a solid in terms of iterated triple integrals in cylindrical coordinates. Do NOT evaluate these integrals.

5. (14 points) Find an equation for the tangent plane to the surface $z = x^2 - y^2$ at the point $(3, 1, 8)$. Also find parametric equations for the normal line to the surface at this point.

6. (18 points) Find all critical points of

$$f(x, y) = x^2 + 2y^2 + x^2 y$$

and classify them as local maxima, local minima or saddle points.

7. (16 points) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x + y^2$ on the ellipse $x^2 + 2y^2 = 4$.

8. (16 points) Evaluate $\int_0^1 \int_y^1 \left(\sin x^2\right) \, dx \, dy$ by interchanging the order of integration.

9. (14 points) Given that $g(t) = f(x(t), y(t))$, where $f_x(1,2) = -1$, $f_y(1,2) = 3$, $x = e^{2t}$ and $y = t^2 + 2$, compute $g'(0)$.

10. (14 points) Sketch the solid Q bounded by $x^2 + y^2 + z^2 = 4$ and above $z = 1$ and express the triple integral

$$\iiint_Q \left(x^2 + y^2 + z^2\right) dV$$

as an iterated integral in spherical coordinates. Do NOT work the integral out.

11. (16 points) Let $F = xyz\mathbf{i} + \left(z^2 + x\right)\mathbf{j} + xz^2\mathbf{k}$ and S be the portion of $z = 1 - \sqrt{x^2 + y^2}$ above the xy-plane, with upward pointing normal. Use Stokes' Theorem to evaluate flux integral $\iint\limits_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$.

12. (14 points) Let $\mathbf{F}(x, y) = \left(2xy + y^2\right)\mathbf{i} + \left(x^2 + 2xy + y^3\right)\mathbf{j}$. Without explicitly attempting to find a potential function, show that the vector field $\mathbf{F}$ is conservative. Then find a potential function for $\mathbf{F}$.

13. (14 points) Use the Gauss Divergence Theorem to compute $\iint_{\partial Q} \mathbf{F} \cdot \mathbf{n}\, dS$, where $\mathbf{F} = (2xy + 4x)\,\mathbf{i} + \left(yz - y^2\right)\mathbf{j} - \frac{z^2}{2}\mathbf{k}$ and Q is the solid bounded by a sphere of radius 2 centered at the origin.

14. (16 points) Let S be the portion of the surface $z = x^2 + y$ over the unit square $0 \le x \le 1$, $0 \le y \le 1$ with upward pointing normal and let $\mathbf{F} = \langle xy,\, xyz,\, yz\rangle$. Express the flux integral $\iint\limits_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ as an iterated integral not involving vectors. Do NOT work the integral out.