Exam 6 **Math 208** **Fall 2004**

Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is required.

(16) **1.** Let $\mathbf{F} = \langle 2x, 2yz^2, 2y^2z \rangle$.

(a) Show that $\mathbf{F}$ is conservative without actually finding a potential function for $\mathbf{F}$.

(b) Calculate $\bigtriangledown \cdot \mathbf{F}$.

(12) **2.** Let $f(t)$ be a scalar function, $\mathbf{r} = \langle x, y \rangle$ and $r = \|\mathbf{r}\| = \sqrt{x^2 + y^2}$. Show that $\bigtriangledown f(r) = f'(r)\dfrac{\mathbf{r}}{r}$.

(24) **3.** Use the Divergence Theorem to evaluate $\int\int_S \mathbf{F}\cdot\mathbf{n}dS$, where $\mathbf{F} = \langle y^3 - 2x, e^{xz}, 4z\rangle$ and S is the boundary of the rectangular box $0 \le x \le 2$, $1 \le y \le 2$, $-1 \le z \le 2$, with exterior unit normal.

(24) **4.** Use Stokes' Theorem to evaluate $\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = \langle \sin(x^2), y, z - y\rangle$ and curve C is the horizontal triangle from $(1, 0, 2)$ to $(1, 1, 2)$ to $(0, 0, 2)$ in that order.

(24) **5.** Let a surface be given by $z = \sqrt{x^2 + y^2}$ where $(x, y) \in R = \{(x, y) \,|\, 0 \leq x \leq 1,\ 0 \leq y \leq 2\}$.
(a) Find formulas for vector and scalar differential surface area $d\mathbf{S}$ and dS in terms of dA, differential surface area in the xy-plane.

(b) Express $\iint_S f(x, y, z)dS$ as an iterated integral in x and y where $f(x, y, z) = \sin(xyz^2)$. Do *not* work the integral out.

(c) Express $\iint_S \mathbf{F} \cdot d\mathbf{S}$ as a double integral over R, where $\mathbf{F} = \langle x, 0, y^2 z \rangle$. Do *not* work the integral out.