Exam 5 **Math 208** **Fall 2004**

Name:____________________________ **Score:**____________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is required.

(24) **1.** A vector field is given by $\mathbf{F} = \langle 2y, 3x^2 \rangle$.

(a) Is $\mathbf{F}$ conservative? Justify your answer.

(b) Find equations for all flow lines for the vector field $\mathbf{F}$.

(24) **2.** Let $\mathbf{F}(x,y) = \dfrac{1}{x^2+y^2}\langle -y, x \rangle = \bigtriangledown \arctan(y/x)$. Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is the unit circle with center at the origin, oriented clockwise.

(20) **3.** Let $F = \langle 3x^2y^2, 2x^3y - 4\rangle$. Show that the line integral

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \oint_C 3x^2y^2\,dx + \left(2x^3y - 4\right)dy$$

is path independent and use the fundamental theorem for line integrals to evaluate this integral if C runs from $(1,2)$ to $(-1,1)$. (You should find a potential function.)

(20) **4.** The closed curve $x^{2/3} + y^{2/3} = 1$ is parametrized by the equations $x = \cos^3 t$ and $y = \sin^3 t$, where $0 \le t \le 2\pi$. Use this fact and Green's theorem to express the area of the region R bounded by this curve as a definite integral in t. *Do not evaluate the integral.*

(12) **5.** Evaluate

$$\oint_C \left(e^{x^2} - 2y\right)\, dx + \left(e^{y^2} + 4x\right) dy$$

where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise. (Green's theorem might help here.)