

Math 208
Cylindrical and spherical coordinates problems

Set up and evaluate each of the following in either cylindrical or spherical coordinates, whichever is more appropriate:

1. $\int_Q x \, dV$, where Q is the region with $x \geq 0$, inside the sphere $x^2 + y^2 + z^2 = 16$, and below the cone $z = \sqrt{x^2 + y^2}$.

2.
$$\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \int_{-2\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}} \frac{x}{x^2+y^2} \, dz \, dy \, dx$$

3.
$$\int_0^2 \int_0^{\sqrt{8-2y^2}} \int_y^{\sqrt{8-y^2-z^2}} \frac{1}{\sqrt{x^2+y^2+z^2}} \, dx \, dz \, dy$$

4.
$$\int_{-2}^0 \int_{2y^2}^{-4y} \int_{-y}^{\sqrt{z-y^2}} \frac{1}{x^2+y^2} \, dx \, dz \, dy$$

5. Find the mass and z -coordinate of the center of mass of the object inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$, if the density is $\delta(x, y, z) = \frac{1}{x^2+y^2+z^2}$.

Answers: 1) $48\pi + 32$

2) $\frac{32}{3} = 10\frac{2}{3}$

3) π

4) $4\pi - 2$

5) mass = 3π , $\bar{z} = 0.75$