

Name: _____

Score: _____

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate*. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

(30) 1. Let $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4] = \begin{bmatrix} 1 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \\ 4 & 8 & 2 & -2 \end{bmatrix}$ with reduced row echelon form $R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathcal{R}(A)$, the row space of A .

(b) Find a basis for $\mathcal{C}(A)$, the column space of A .

(c) Find a basis for $\mathcal{N}(A)$, the null space of A .

(d) Find all possible linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ that sum to $\mathbf{0}$.

(e) Which $\mathbf{v}_j$'s are redundant in the list of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$?

(f) Find a basis of $\mathbb{R}^3$ containing a basis of $\mathcal{C}(A)$.

(16) **2.** Use the Subspace Test to decide if W is a subspace of the vector space V , where

(a) $V = \mathbb{R}^3$ and $W = \{(a, b, a - b + 1) \mid a, b \in \mathbb{R}\}$

(b) $V = C[0, 1]$, the continuous functions on $[0, 1]$ and $W = \{f(x) \mid f(x) \in C[0, 1] \text{ and } f(1) = 0\}$.

(10) **3.** Assume that $1 + x, x + x^2, 1 - x$ is a basis of $\mathcal{P}_2$, the space of polynomials of degree at most two, and find the coordinates of $2 + x^2$ relative to this basis.

(8) **4.** Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Find the adjoint matrix $\text{adj}(A)$ of A .

(12) **5.** You are given that $\mathbf{w}_1 = (0, 1, 0)$, $\mathbf{w}_2 = (1, 1, 1)$ is a linearly independent set in $V = \mathbb{R}^3$ and $\mathbf{v}_1 = (1, 3, 1)$, $\mathbf{v}_2 = (2, -1, 1)$, $\mathbf{v}_3 = (1, 0, 1)$ is a basis of V . Steinitz substitution says that $\mathbf{w}_1$, $\mathbf{w}_2$ can be substituted into the basis in place of certain $\mathbf{v}_i$'s. Which substitutions work?

(16) **6.** Fill in the blanks or answer True/False (T/F).

(a) Every vector space is finite dimensional (T/F) _____.

(b) Elementary row operations on a matrix do not change the column space (T/F) _____.

(c) If $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ are both vector solutions to the linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_0$ is in the null space of A . (T/F) _____.

(d) The function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by $T((x, y)) = (x + y, x - 2y)$ is linear (T/F) _____ and one-to-one (T/F) _____.

(e) The Basis Theorem asserts that every finite dimensional vector space _____.

(f) The Dimension Theorem asserts that _____.

(g) A basis of a vector space is by definition _____.

(10) **7.** (a) Show that the columns of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ form a linearly dependent set.

(b) (*Honors students only*) Prove that any set of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ in a vector space V that contains the zero vector is a linearly dependent set.