Exam 2 **Linear Algebra** **Sample**

Name:______________________________ **Score:**______________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculators are allowed.

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(30) **1.** Let $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4] = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \end{bmatrix}$ with reduced row echelon form $R = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathcal{R}(A)$, the row space of A.

(b) Find a basis for $\mathcal{C}(A)$, the column space of A.

(c) Find a basis for $\mathcal{N}(A)$, the null space of A.

(d) Find all possible linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ that sum to $\mathbf{0}$.

(e) Which $\mathbf{v}_j$'s are redundant in the list of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$?

(f) Find a basis of $\mathbb{R}^3$ containing a basis of $\mathcal{C}(A)$.

(21) **2.** Use the Subspace Test to decide if W is a subspace of the vector space V, where
(a) $V = \mathbb{R}^3$ and $W = \{(a, 0, 1) \mid a \in \mathbb{R}\}$.

(b) $V = C[0, 1]$, the continuous functions on $[0, 1]$ and W is the set of $f(x) \in V$ such that $f(0) = 0$.

(c) $W = \{\mathbf{v} \in V \mid T(\mathbf{v}) = 0\}$ where $T : V \to U$ is a linear operator (you may assume that $T(0) = 0$.)

(10) **3.** Show that $1 + x$, $x + x^2$, $1 - x$ is a basis of $\mathcal{P}_2$, the space of polynomials of degree at most two, and find the coordinates of $2 + x^2$ relative to this basis.

(15) **4.** You are given that $\mathbf{w}_1 = (0, 1, 0)$, $\mathbf{w}_2 = (1, 1, 1)$ is a linearly independent set in $V = \mathbb{R}^3$ and $\mathbf{v}_1 = (2, -1, 1)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (1, 3, 1)$ a basis of V. Steinitz substitution says that $\mathbf{w}_1$, $\mathbf{w}_2$ can be substituted into the basis in place of certain $\mathbf{v}_i$'s. Which substitutions work?

(14) **5.** Fill in the blanks or answer True/False (T/F). Each correct answer is worth 2 points, false answer worth -1 points and blank answer worth 0 points for a minimum of 0 and maximum of 14:

(a) Every vector space is finite dimensional (T/F) __________.

(b) Elementary row operations on a matrix do not change the column space (T/F) __________.

(c) If $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ are both vector solutions to the linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_0$ is in the null space of A. (T/F) __________.

(d) The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (x + y, x - 2y)$ is linear (T/F) __________.

(e) The Basis Theorem asserts that every finite dimensional vector space ____________________.

(f) The Dimension Theorem asserts that ____________________.

(g) A basis of a vector space is by definition____________________.

(10) **6.** Let U and V be subspaces of the finite dimensional vector space W such that $U \cap V = \{0\}$. Show that $\dim(U + V) = \dim U + \dim V$. (Hint: show the union of bases of U and V is a basis of $U + V$.)