Course Assignments for JDEP 384H

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Note: Unless otherwise stated, it is always permissible to use Matlab for calculations. As a general rule, you are expected to show your work. In particular, if you use Matlab to solve a problem, you are expected to provide a transcript via diary or cut and paste into a document. Do NOT include pages of output, only that which is relevant to you solution – use the semicolon to supress unnecessary output. Typed documents are preferred; hand written copy will be accepted if it is *neatly* written. Unless otherwise stated, hardcopy is the rule. Problems that are to be worked by individuals without collaboration will be marked "(I)". Each assignment will be worth 45 points.

Assignment 1

Points: 45

Due: January 25

(1) Let

$$A = \begin{bmatrix} 3 & -1 & 0 & 2 \\ 1 & 0 & 2 & 4 \end{bmatrix} \text{ and } x = [1, -2, -1, 3].$$

Perform the following calculations by hand (use Matlab to check your work, if you're not sure of the answers) or explain why they are impossible:
(a) $A\mathbf{x}^T$ (b) AA^T (c) $AA^T\mathbf{x}^TA$
(d) $A + I_2\mathbf{x}$ (e) I_2A (f) $\|\mathbf{x}\|_2$
(g) $\|\mathbf{x}\|_1$ (h) $\|\mathbf{x}\|_\infty$

(2) Consider the following linear system:

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ x_1 + x_2 &= 2 \\ 2x_1 - x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 5 \end{aligned}$$

(a) Exhibit the coefficient matrix A and right hand side vector $\mathbf{b}$ for this system, so that if $\mathbf{x} = (x_1, x_2, x_3, x_4)$, then the system is given by $A\mathbf{x} = \mathbf{b}$. **Note:** I use the convention that a vector given by soft parentheses represents a column vector. Therefore

$$(x_1, x_2, x_3, x_4) = [x_1, x_2, x_3, x_4]^T = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}.$$

(b) Get help on the Matlab backslash command \ and use it to solve $A\mathbf{x} = \mathbf{b}$.
(c) Compute the residual vector $\mathbf{b} - A\mathbf{x}$. Is it the zero vector?

(d) Write out explicitly the linear system defined by the matrix equation

$$A^T A\mathbf{x} = A^T\mathbf{b}.$$

(e) Calculate the norm of the residual

$$\mathbf{r} = A^T\mathbf{b} - A^T A\mathbf{x}$$

where $\mathbf{x}$ is the vector obtained in (b). Conclusions?

(3) Recall (or accept) that a plane rotation matrix is defined by the formula

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}.$$

If $\mathbf{x} = (x, y)$ represents a point in the plane, then $R(\theta)\mathbf{x}$ represents the point obtained by rotating $\mathbf{x}$ counterclockwise about the origin by an angle of θ radians.
(a) Use Matlab to plot the function $y = x\cos^2(2\pi x)$ on the interval $[-\pi, \pi]$.
(b) Set hold on and superimpose plots of this curve rotated by both $\pi/6$ and $\pi/2$ radians. (Hint: it helps to form the matrix $[x; y]$ whose columns are points of the plot. Then multiply by the appropriate rotation matrix. The new rows are what you should plot.)

(4) Let

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 4 & 3 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

(a) Think of A as a transition matrix for a discrete dynamical system and find the state vector $\mathbf{x}^{(20)}$, starting with a random initial vector (use Matlab's rand command). Then calculate $\mathbf{x}^{(20)}/\left\|\mathbf{x}^{(20)}\right\|_2$.
(b) Next use Matlab's eig command to compute an eigensystem for A (eigenvectors plus eigenvalues).
(c) Comment on any connections you see between parts (a) and (b).

(5) Let $f(x) = \frac{1}{1+x^2} + \cos(2x)$ be defined on the interval $[0, 6]$. In this exercise you will be asked to create multiple plots together in a single figure.
(a) Compute the function $f'(x)$.
(b) Create a figure with four subplots. To find out how to do so, read the last page of the file MatlabLecture-384H.pdf.
(c) Use the forward difference $(f(x+\Delta x) - f(x))/\Delta x$ as an approximation to $f'(x)$ and insert into the figure plots of the difference $(f(x+\Delta x) - f(x))/\Delta x - f'(x)$ on the interval $[0, 6]$ using $\Delta x = 0.2$ and $\Delta x = 0.1$.
(d) The centered difference $(f(x+\Delta x) - f(x-\Delta x))/(2\Delta x)$ is supposed to be an improved approximation to the derivative. Repeat (c) with entered difference in place of forward difference and comment on the differences in the graphs that you see.

(6) Consult the common distributions subsecion of the file ProbStatLecture-384H.pdf.
(a) Confirm the approximation assertion about Poisson vs binomial by calculating certain values, say with $\mu = 3, 4, 5$.
(b) Confirm the limiting assertion about the Student's t distribution by creating plots (in a single figure) of various t distributions and a standard normal distribution.

Assignment Closed

Assignment 2

Points: 45
Due: February 15

(1) In this exercise you are given that three securities S_1, S_2, S_2, have expected rates of return $\mu_1 = 0.1$, $\mu_2 = 0.15$ and $\mu_3 = 0.2$, respectively. The standard deviation of the securitites are $\sigma_1 = 0.28$, $\sigma_2 = 0.24$ and $\sigma_3 = 0.25$, respectively. The correlations between these rates of return are $\rho_{12} = -0.1$, $\rho_{23} = 0.2$ and $\rho_{31} = 0.25$. Define the vector random variable $\mathbf{S} = (S_1, S_2, S_3)$. You may find it helpful to review the section in ProbStatLecture-384H on vector random variables.
  (a) Find the mean vector and covariance matrix for $\mathbf{S}$.
  (b) If one owns a portfolio which is 50% invested in S_1 and 25% in each of the remaining two, what is the expected return and standard deviation of this portfolio?
  (c) What is the weighting of the portfolio that yields the maximum expected return? What is the corresponding risk (as measured by the standard deviation of this portfolio)?
  (d) Find a weighting with no short positions that is also an eigenvector of the covariance matrix of S. What are the risk (as measured by variance) and expected return of this portfolio?

(2) The data in the text file TreasuryRates2003 represents the weekly values of the 3-Month, 6-Month, 1-Year, 2-Year and 3-Year TCM (Treasury Constant Maturity) notes, one row per week. You can obtain this variable by the Matlab command "`load TreasuryRates2003`", assuming this data is in your current working directory.
  (a) View each row as a sample from the 5-vector random variable $\mathbf{X}$ and use Matlab to compute the mean and covariance matrix of $\mathbf{X}$. (Use commands `cov` and `mean`.)
  (b) Make a plot of the means of the rates of the notes against maturity (in years). What does this term structure suggest to you about the future market?
  (c) Find the correlation matrix for $\mathbf{X}$ and find the notes that have the strongest correlation and weakest correlations.
  (d) Treat the mean and variance for X_5 (3-Year notes) over the year's data as the true mean and variance. Now compute confidence intervals for the true mean using the the first 26 weeks of data, assuming (i) known variance and (ii) unknown variance, and also find a confidence interval for the true variance. You may assume the samples are i.i.d. random normal variables. Do your intervals contain the "true" mean and variance?

(3) Refer to Lecture 9 notes for the Wyndor Glass Company problem.
  (a) Solve this problem graphically.
  (b) Solve it using the Matlab programs `linprog` and `lp`.
  (c) Suppose the profit per batch is changed from \$3K, \$5K to \$3K, \$2K. Solve this problem graphically. How does it differ from the solution in part (a)?
  (d) Suppose management insists that Plant 3 must run at full capacity (no slack time). Solve this modified problem graphically.

(4) The future value S_1 of a stock in one time period with present value $S_0 = 100$ is modeled by a binomial lattice with multiplicative shocks of $u = 1.2$ and $d = 0.8$. Probability of up is p and down is $1 - p$, but p is not known. A zero-coupon bond

with face value $B_0 = 100$ earns risk-free interest 10% in the same time period. You construct a weighted portfolio of x stocks and y bonds (short positions in either asset are allowed.)

(a) What is the value of your portfolio in terms of x and y in one time period in the cases that the stock goes up or down?
(b) What is the value of a call option (at strike price S_0) on the stock in one time period in the cases that the stock goes up or down?
(c) Match the values of (a) and (b) in the cases that the stock goes up or down, and solve the resulting system of equations for x and y. What is the present price of this portfolio?
(d) With the standard market assumptions, the present price of this portfolio is exactly the present market price of a call option on the stock. Explain why.

(5) Suppose that you have the same objective and tools as in the example on page 14 of Lecture 8, except that you have additionally a three year coupon bond C with annual coupons of 7.5% and face value of \$100. Also, you disallow short positions in any bond. For your convenience, a modification of `ImmuneDur.m` with name `Exercise2_5.m` is to be found in our home directory.
(a) Is there a weighting that matches weighted durations and convexities to the duration and convexity of the target cash flow?
(b) Find the weighted combination of the three bonds that matches duration and maximizes the convexity of the portfolio.
(c) Find the weighted combination of the three bonds that matches duration and maximizes the weighted coupon rate of the portfolio.
(d) Calculate the displacement from your objective if the prevailing interest rate rises or falls 3% after one year for each viable stratgegy above. Which is superior in this case?

Assignment Closed

Assignment 3

Due: March 20

(1) You are designing a portfolio consisting of the three assets described in Exercise 1 of Assignment 2.
(a) Express the problem of finding the minimum risk portfolio as a quadratric programming problem and use the MatlabTools program `quad_prog.m` to find the minimum risk. (Be sure to grab the latest copy of this program from our home directory in MatlabTools for this exercise.)
(b) Use `quad_prog.m` to make a graph of the mean-variance efficient frontier of this portfolio using steps of 0.2% from minimum to maximum return.

(2) You are selling the portfolio of Exercise 1 to a client who is leery of asset 3 and does not want more than 40% of the portfolio in that asset.
(a) Graph an efficient frontier for possible portfolios for this client.
(b) Your client has a target of 17% expected return with, of course, minimum risk. Your advice?

(3) Refer to the example portfolio of two assets whose efficient frontier we graphed in Lecture 11. Since there are only two weights, the value of one weight, say $w = w_1$, uniquely determines the portfolio since $w_2 = 1 - w_1$. Make a plot on a single graph

of the value at risk of the portfolio as a function of w for time horizons of 50 and 100 days. Assume the variances and in the example are annual, so prorate them for daily rates. Also assume that the value of the portfolio in all cases is \$100.

(4) Give a careful explanation why Ito's Lemma applied to $f(S) = \log S$ with random process $\frac{dS}{S} = \sigma\, dW + \mu\, dt$, leads to the formula

$$df = \sigma dX + \left(\mu - \frac{1}{2}\sigma^2\right) dt.$$

How does this compare this to what you get by applying the deterministic chain rule to obtain a differential formula for df?

(5) Consider the random walk suggested by the differential equation $dS = \sigma S dX + \mu S dt$ with $\mu = 0.1$ and $\sigma = 0.3$ and time in units of years.
  (a) Create a graph with 10 simulations of this random walk, starting with $S(0) = 100$, and in steps of $dt = 1/12$ over a two year period. Before you begin, reset the random number generator of Matlab with the command `randn('state',0).`
  (b) Calculate $S(2)$ by running 100 simulations of this walk and compute the mean and variance of this sample. How does the sample mean and variance compare with the formula at the bottom of page 99 of the text and the volatility, respectively?

(6) Consider a call option with strike price of 50 on a stock at a time that is 5 months before expiry. Assume that the volatility of this stock is $\sigma = 0.4$ and that the risk-free interest rate is 10%.
  (a) Make a graph of the payoff curve and the price of the option for S ranging from 20 to 100, assuming that the call is European (use `bseurcall.m`.)
  (b) On the same graph as (a) plot the price of the option assuming that it is American (use `LatticeAmCall.m`, lattice of 100 time steps, and a for loop.) Comment on the plots.
  (c) Repeat (a) and (b) under the assumption that the stock pays dividends at a continuous rate of $D_0 = 0.06$.

Points: 45
Due: March 8

Assignment Closed

Assignment 4

Points: 45
Due: April 5

(1) In Matlab create an anonymous function $f(x)$ with the formula

```
f = @(x) x.^4 - 4*x.^3 + 6*x.^2 - 4*x + 1
```

  (a) What function does this represent and what is its derivative?
  (b) Plot this function on the interval $[1 - dx, 1 + dx]$ in steps of $dx/100$ with $dx = 0.001$ and also $dx = 0.0001$. Are the graphs reasonable? Explain.
  (c) Find the value of k such that using $h = 10^{-k}$ gives the best approximation to $f'(0)$ using forward differences and calculate the relative error of the approximation.
  (d) Same question as (c) using centered differences.

(2) Consider the system

$$\begin{aligned} 4x_1 + 3x_2 + 3x_3 &= 5 \\ 3x_1 + 4x_2 + 3x_3 &= 3 \\ 3x_1 + 3x_2 + 4x_3 &= 2. \end{aligned}$$

(a) Find the solution to this system.
(b) Exhibit the iteration matrices for the Jacobi and Gauss-Seidel methods.
(c) Find the eigenvalues of matrix of (b) and indicate which method will converge.
(d) Find the number of iterations of the convergent method that are required to reduce the absolute error to below 0.01, starting with the zero solution.

(3) Let $\mathbf{x} = (x_1, x_2)$ and $F(\mathbf{x}) = \left(x_1^2 - 3x_2^2 + 3, \sin\left(\frac{\pi}{12}x_1x_2\right) + 1\right)$.
(a) Use Matlab's fminsearch function to find a solution to $F(\mathbf{x}) = \mathbf{0}$ as we did in an optimization example calculation.
(b) Assume that $x_1x_2 = -30$, and use this to eliminate x_1 from the first coordinate of F. Use fzero to find a zero of this first coordinate as a function of x_2 and evaluate F at the resulting (x_1, x_2).

(4) Refer to the European put option of Problem 2 of the take-home midterm. Make a plot of the difference between the true value of the option on the interval $20 \le S \le 80$ at $t = 0$ and the approximation to this curve that you obtain by creating a cubic spline with Matlab that interpolates the curve at the points $S = 20, 30, 40, 50, 60, 70, 80$. What is the largest error on this interval?

(5) Let $f(x) = e^{-x^2/2}/\sqrt{2 * \pi}$, the probability density function for the standard normal distribution. Let $I = \int_0^2 f(x)\,dx$.
(a) Compute an accurate value to I by using statistical functions in Matlab and confirm it with Matlab's quad function.
(b) Approximate the integral using 100 function evaluations and the trapezoidal method.
(c) Approximate the integral using Gaussian quadrature and 8 evaluations.
(d) Approximate I using 1000 evaluations and the hit or miss Monte Carlo method.
(e) In each of (b)-(d), calculate cost (number of function evaluations needed) per correct digit of the answer.

Assignment Closed

Assignment 5

Points: 45
Due: April 25

(1) For this exercise, use the data you can load from the file AsianExampleData5_1 which can be found in the directory Week15. You are to price an Asian call with the parameters in this file. Follow the Asian call example in Lecture 24.
(a) Identify the approximation to the price and corresponding confidence interval of the option with these parameters, using the simple Monte Carlo method of Lecture 24.
(b) Use the price of a European call with the same parameters as a control variate and calculate the approximation to price and confidence interval. Is there any improvement over (a)?

(2) Modify the down-and-out put example of Lecture 24 to approximate the value of an up and out call with the same data as in AsianExampleData5_1.m and an up barrier of $S_b = 125$. Find an estimate of the lowest value of S_b that puts the value of this option within 90% of the value of the corresponding call without a barrier.

(3) Consider the game with payoff table

Strategy		Player 2			
		1	2	3	4
Player 1	1	3	-2	1	1
	2	0	2	1	0
	3	1	0	-2	2

(a) Does dominated strategy elimination or minimax/maximin yield a solution to the game?

(b) Solve this game with Matlab. (It would be more elegant to write a program with the syntax `[x,y,p] = gamesolve(A)`, where A is the payoff table, x, y, p the optimal strategies and payoff, but this is not required.)

(c) Is this a fair game? If not, what shift in payoffs would make it fair?

(4) A credit manager in the small business division NMO bank approves (or rejects) credit lines for small businesses. A customer is requesting a credit line of $100,000 for her specialties clothing store. Experience shows that 20% of customers in this category are poor risks, 50% are average and the rest are good risks. If credit is extended, the average profit is -15% for poor risks, 10% for average risks and 20% for good risks. For $4,500 the credit manager could obtain additional information from a credit-rating company whose track record with the bank is as follows: If a customer turns out to be a poor risk, the company rates him as poor 45% of the time and average in 35% of the time. If the customer is an average risk, the company rates him average 60% of the time and poor 35% of the time. If the customer is a good risk, the company will rate him as good 50% of the time and average 30% of the time.

(a) Write a decision analysis form for this problem by identifying the states of nature, alternatives, prior probabilities, payoff table and conditional probability table of the credit-rating firm.

(b) Use Bayes' decision rule to determine which course of action should be taken if the credit-rating firm is not used.

(c) Find the EVPI and use it to decide whether or not to seek additional information from the credit-rating firm.

(d) Compute the probabilies that the credit-rating company will rate a customer as a poor, average or good credit risk.

Assignment Closed