A "Wet-Lab" Calculus for the Life Sciences†

James L. Cornette[1], Department of Mathematics, Iowa State University
Ralph A. Ackerman, Department of Ecology, Evolution and Organismal Biology, Iowa State University
Brin A. Keller, Department of Mathematics, Michigan State University
Gail B. Johnston, Department of Mathematics, Iowa State University

Name of Institution	Iowa State University
Size	31,000 students (25,500 undergraduate and 5500 graduate)
Institution Type	Land grant university, comprehensive undergraduate and graduate programs
Student Demographic	Freshman and sophomore life sciences majors take this course or the engineering calculus course
Department Structure	Mathematics and several life sciences departments are in the College of Liberal Arts and Sciences

Abstract

In 1994, faculty at Iowa State University introduced a calculus and differential equations course for life sciences students, the unique characteristic of which was an associated wet-lab laboratory. The students gathered data from living systems and simulations of living systems, which they analyzed using difference equations, differential equations, derivatives, or integrals. Here we describe steps that we took to introduce and develop a two-semester calculus and difference and differential equations course for biology students, and we discuss the value of biological data in such courses. We also discuss the change in course format to large lecture and no laboratory as faculty who introduced the course left the university and extreme financial restrictions were placed on universities during the first decade of the 21st century.

Course Structure

- Weeks per term: Two 15-week semesters
- Classes per week/type/length: Four 50-minute lectures per week
- Labs per week/length: Seven 2-hour laboratories per semester
- Average class size: 34 students

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[1] cornette@iastate.edu

- Enrollment requirements: Competence in high school algebra, geometry and trigonometry

- Faculty/dept per class, TA's: Mathematics faculty teach the lectures; Biology teaching assistants teach the laboratory with biology faculty supervision

- Next Course: This course and possibly a semester of statistics constitute the mathematics taken by these students

- Website: The textbook for the course is posted on http://cornette.public.iastate.edu/CLS.html

- A print version of the text is now available from CreateSpace, a subsidiary of Amazon

Introduction

In 1994 conditions were good for introducing a calculus for the life sciences course at Iowa State University, a university of 28,000 students that is very strong in science and technology. Interdisciplinary studies were encouraged, the life sciences faculty were becoming aware of the need for more mathematics, statistics and computer science in their own work and for their students, weekly seminars in computational and mathematical biology had been held for six years, and there were mathematicians who had published in biological journals and a biologist who used mathematics in his research and was auditing mathematics courses. There was strong administrative support and support from the National Science Foundation.

Our job was to develop the content of the course. We began with the (correct) premise that life sciences students would respond well to mathematics that grew out of life science laboratories and data from their literature. There was no lab manual or a text coordinated with laboratories, but we said we would write them. That turned out to be a large task, however, and only in May of 2005 was a text based on the course delivered to a publisher. The text is now posted on the web at http://cornette.public.iastate.edu/CLS.html, and a hard copy is available in two volumes (Cornette and Ackerman, 2011, 2012).

Discussion

In the first years, the main task was to plan and write material; classes and laboratories were often preceded by a quick trip to the university printing office. Cornette (a mathematician) wrote the text and Ackerman (a biologist) organized the laboratories and wrote lab material. After most of the text had been developed and there were some 150 students, a major impediment was copyright permission to use some of the data from the literature. The most extreme was the cost of $2 per student copy for two figures from *Scientific American*. They are excellent figures but we excluded them.

We began with four one-hour class meetings per week and seven or eight three-hour laboratories per semester. It is difficult to add a laboratory to the schedules of life sciences students, and after two years we moved to a two-hour laboratory, one hour of which replaced one of the four one-hour class meetings. In this arrangement, some bacterial growth laboratories were run in relay. One group of students inoculated a bacterial culture and monitored the first hour of growth and later growth was monitored by other students.

The laboratories illustrated

- exponential and logistic growth of bacteria,
- quadratic growth of mold,
- exponential decay of light intensity with depth in a lake,
- reciprocal of distance squared and reciprocal of distance decrease of light intensity in air,
- one- and two-compartment models of pharmacokinetics,
- Torricelli's quadratic decrease with time of water flow from the bottom of a reservoir,
- speed of falling objects and harmonic oscillations,
- Fick's law used to measure cardiac output and the approximately cubic decrease in the weight of a melting ice ball,
- Newton's law of cooling,[2]
- solar radiation,
- cooling and dehydration of an egg,
- simulated breaking of whelk by crows,
- cricket chirp rate dependence on temperature,
- air compression in a syringe,
- areas of leaves, and
- butyl chloride decomposition rate.

Light depletion with increasing water depth and simulated breaking of whelk by crows have been described in (Keller 1998) and (Keller and Thompson 1999). These and simulation of cardiac output, are included in the National Council of Teachers of Mathematics website, Illuminations (Keller and Thompson 2012a,b,c).

An enzyme kinetics lab led towards the important Henri-Michaelis-Menton equation, but was sufficiently difficult that we used it only twice. We were not

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[2]We did not use Torricelli, Fick, and Newton in discussion with the students.

successful in developing a laboratory on diffusion in a medium (diffusion across a membrane is easy) and would welcome suggestions for one.

The students also were assigned six-week projects in which they could either find a data set from their respective majors or select a project from a list that we provided. Many of the students did find data from their own major, and it is not a surprise that their projects were outstanding. Students submitted written reports and presented verbal reports to the class. The final examination included questions related to the projects. As with the lab reports, the projects were done by groups of three students. We had 34 - 38 students per lecture section and 16 (better) and sometimes 18 students in the laboratory.

The students appreciated and enjoyed the laboratories and ownership of the data. Statistical analysis of messy real life data and the discussion of residuals were important experiences for the students. We tried to find appropriate biological models to use for homework assignments, classroom examples, and exams, but finding them was not easy.

Laboratory reports descriptive of the data and the students' proposed mathematical models of the underlying process served to solidify the students' understanding of the connection between the biology and the mathematics. Students were motivated by the laboratories, and the mathematics was perhaps more easily understood, but the algebraic operations were not easier for them. Group laboratory reports were assigned to keep the evaluation manageable. Many of the reports were well written and reports that were poorly written were returned for improvement.

It is our hypothesis that the goal of calculus in the life sciences is primarily modeling living systems with difference equations and differential equations. Understanding the concepts of derivative and integral is crucial, but the ability to compute many specific derivatives and integrals is not important. The derivative and integral were carefully defined, however, and distinguished from the discrete data of the laboratories. The analysis of calculus thus extends the discrete laboratory observations. The Fundamental Theorem of Calculus was clearly explained analytically, as it did not seem suited for a laboratory exercise.

We also included some not necessarily biological applications of the derivative and integral (max-min, related rates, area, volume, surface area problems). For example,

1. A 2-meter tall vertical fence stands 1 meter away from a 5-meter vertical wall. What is the shortest ladder that will reach across the fence from the ground to the wall and just touch the top of the fence? Assume the ground to be horizontal.

2. Find the centroid of the right circular cone with base radius *r* and height *h*.

It was correctly argued (Mathematical Biosciences Institute Curriculum Workshop 2007) that Problem 1 is not of much interest to the life sciences students. Few students in any calculus class find the problem entertaining. An alternate, 'What length ladder can be carried horizontally around a 90º turn from a 2 meter wide hallway into a 1 meter wide hallway?' usually is met with only slightly greater interest. But both are clearly defined exercises where students identify relevant variables, find relations between them, and obtain a function of a single variable whose extrema give a solution. Until the life sciences provide a rich supply of such problems, we should continue to use the geometry problems that have proved over many years to be useful to students in gaining an understanding of calculus.

After three years, the course had grown to four sections in the fall and two or three in the spring. It was beyond our ability to teach all of the sections, and some faculty and graduate students who were sympathetic, but without biology experience, filled in quite well. We were fortunate to have several laboratory assistants from biology who had taken or were taking serious mathematics. Most of the mathematics faculty and teaching assistants, however, found it very time consuming to grade lab reports and semester projects and to make good use of the laboratory data. It is easier to teach engineering calculus.

Laboratories are an expensive add-on to a mathematics course. The chair of zoology initially arranged with the college dean for funding of laboratory assistants and made available a 16-student laboratory space, and continued support after our NSF grant expired. Some equipment had to be scrounged, but generally the laboratories were well equipped. The students responded well, wrote substantial laboratory reports and grasped the mathematics emerging from the laboratories. The biggest laboratory difficulty was that bacteria do not always grow, even when properly stimulated, so the resulting growth curve may be a horizontal line. Furthermore, the butyl chloride decomposition curve was not the simple exponential decay that is illustrated in the students' chemistry text. The real world does not always conform to our expectations.

The laboratory is not now being offered and the class is taught as lectures to groups of 100 to 120 students. The reasons for discontinuing the laboratories were budget reductions that began in 2001 and the retirement or departure from ISU of mathematics faculty wishing to use them.

It was a useful statement to students that the laboratories were held in the building of a biology department. We have found, however, that the laboratory concept can be used even in mathematics lectures to 100 to 120 students. Some of the laboratories can be performed as demonstrations by the lecturer or a group of two or three students in a mathematics classroom. For example, we measure the temperature of an egg (or a pint of water) as 37º C initially and perhaps 34º C and ten minutes later. We ask the students to estimate what the egg temperature will be thirty minutes later. There can be a discussion about how long an egg can safely remain uncovered in a nest during which we hope that a student inquires what the ambient temperature is. Other experiments suitable for large lectures include light decay with depth of water, simulated cardiac output, one- and two-compartment models of pharmacokinetics, water draining from a reservoir, and work done in compressing air.

The project and the participants greatly benefitted from the National Science Foundation Grant number DUE 9354437, for the financial support and especially for the recognition from our university community that it was a worthwhile project.

Suggestions

A calculus for life sciences course requires life science curricula to expand mathematics requirements from none or only three or four semester credits to eight semester credits, which will displace something else from that curriculum. It also requires an expansion of freshman-level mathematics teaching, not highly valued in faculty evaluations. Nevertheless, an outstanding course can change the way students think about the life sciences. Mathematics departments have an inglorious practice of teaching a one-semester three-credit course about calculus for social sciences and business. We will be failures if that becomes the norm for life science calculus courses.

Conclusion

The role of data in life science calculus courses is still under debate. At the Mathematical Biosciences Institute Curriculum Workshop (Mathematical Biosciences Institute 2007), one speaker looked for the first occurrence of a data set in most of the current texts. All of them had at least one data set, with the first occurrence ranging from about page 10 to about page 800. Calculus has been taught for many years without reference to data, so why bother? Our answer (though, without hard data) is that students are more apt to be accepting of a course that includes real data and are more apt to use calculus and to incorporate modeling in their subsequent biological studies.

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