An Interdisciplinary Course, Textbook, and Laboratory Manual in Biomathematics with Emphasis on Current Biomedical Research†

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Name of Institution: Sweet Briar College	
Size	about 750 students
Institution Type	liberal arts college
Student Demographic	mathematics and biology majors and minors with interests in biomathematics
Department Structure	Mathematical Sciences and Biology are separate departments at Sweet Briar College

ABSTRACT

We describe an interdisciplinary course and a textbook in mathematical biology developed by faculty from Sweet Briar College (SBC) and the University of Virginia (UVA) School of Medicine with support from the National Science Foundation (NSF) and the National Institutes of Health (NIH). The course and textbook were designed to supply a bridge between the mathematical and biological sciences at the lower undergraduate level. The topics, except for the introductory material, are based on current research at the UVA School of Medicine and are attractive to mathematics majors who wish to learn how their skills can be applied to solving problems in the life sciences and biology majors who intend to pursue medical or health-related careers. Their modular structure makes the material adaptable to audiences with different backgrounds, needs, and levels of expertise. The course was developed for and is being taught at SBC, but some of the advanced modules are used in a course at UVA for advanced undergraduate and beginning graduate students. Individual modules can be used as stand-alone projects in conventional mathematics and biology courses.

Course Structure

- Weeks per term: 14-week semester
- Classes per week/type/length: three 1-hour lecture periods each week
- Labs per week/length: one 3-hour laboratory periods each week
- Average class size: 7-10 students
- Enrollment requirements: One semester of Calculus, one semester of Biology, and one semester of Statistics.

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- Faculty/dept per class, TAs: Team-taught by one mathematics professor and one biology professor with both doing portions of the lectures and both supervising the laboratory periods.
- Next course: The purpose of this course is to introduce students to the field of biomathematics. There are no specific follow-up courses, but some students will choose biomathematics related projects to satisfy the research requirements of their majors.
- Website: http://www.biomath.sbc.edu

Introduction

Sweet Briar College (SBC), located in central Virginia, is a selective four-year liberal arts and sciences college for women that awards Bachelor of Arts and Bachelor of Science degrees. SBC has approximately 650 students in residence. The average class size is 12 and the student-faculty ratio is 8:1. SBC is consistently cited by the *Princeton Review Guide to the Best 373 Colleges* and *U.S. News & World Report* as one of the nation's leading liberal arts institutions.

Over the past ten years, SBC has implemented extensive science curricular revisions with strong emphasis on faculty-student research that culminates in a Senior Research experience. As required by the general education requirements of the college, emphasis is placed on developing oral and writing skills. The importance of independent research has been recognized by the administration of the college and is a key element of SBC's strategic plan. In the last decade, over three million dollars has been awarded to SBC by the NSF in support of these initiatives, in addition to grants from the NIH and private foundations. Since 2000, an average of 4% of the degrees at SBC has been awarded to mathematics majors, substantially higher than the national average of less than 1% (National Center for Education Statistics, 2010). Biology at SBC has also seen an increase in majors, in line with trends reported by the Department of Education for biological sciences (NCES, 2010) and an increased number of graduates who go on to graduate and professional school.

Yet, when we began this project in 2000,[2] almost nothing in our science curriculum targeted the need for integration of mathematics with biology, a need that all of us doing biomedical research identify as pressing. Because the characterization of biological systems was reaching an unparalleled level of detail, there was clear need to apply tools from mathematics to solve the frontier problems in biology. Modeling of biological systems had also become an important component of experimental work and there was a growing demand for graduates with interdisciplinary training.

Unfortunately, a program to provide such training for our undergraduate students did not exist and, even worse, there was no place in the curriculum where students could see the exciting role of mathematics in biomedical research. The causes of this were easily identifiable.

Too often in traditional math courses, simplistic material is used for applications to the life sciences. These brief examples, often out of context, have little appeal to the students and are often restricted to a few class sessions on word problems having little modeling content. Most mathematics majors complete their degrees with a rich repertoire of mathematical skills but almost no experience in learning how to apply them to problems that arise in science. In spite of the need for integrating mathematics with the life sciences, the emphasis in the mathematics courses is still on techniques rather than on applications.

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[2] This was prior to the release of the report Bio2010 (National Research Council, 2003) that drew national attention to the lack of interdisciplinary interaction between mathematics and biology.

Meanwhile, many of the courses in the biology curriculum make no significant use of mathematical models and theory, regardless of their increasing importance. The students who take these courses have normally completed their major's mathematical requirements (one or two semesters of calculus and a semester of statistics), have memorized a list of algorithms, and have mastered the technical use of some software packages, but have not gained enough conceptual understanding to be able to apply the algorithms in novel situations or to adequately interpret the computer output.[3] As a result, few mathematics majors and biology majors are equipped to use biomathematics techniques required by modern biological research.

In this paper, we describe in detail the undergraduate course Topics in Biomathematics that we have created and now teach at SBC. This course is a first step in integrating mathematics and biology in the undergraduate curriculum.

Course Description

The course is designed for sophomores through seniors and requires prerequisites of one semester of calculus, one semester of statistics, and a one-semester general biology or cell biology course. Our pedagogy focuses on the fundamental concepts, ideas, and biological applications of mathematical tools, and not on rigorous mathematical analysis. We do not attempt systematic presentation of mathematical material, although there are important threads that run through several of the course topics. Instead, by using specialized software (e.g. Berkeley-Madonna, MATLAB, MINITAB, and PULSE_XP) and avoiding most of the mathematical details, our goal is to enable students to experience the usefulness of mathematical techniques applied to biological problems.

Innovative features of the course

Our course differs from other mathematics and biology courses at SBC because

1) It is taught by a biologist and a mathematician to students who have different mathematics and biology backgrounds. Usually, about 50% of the students are biology majors and about 50% are mathematics majors. Each group brings its own expertise into the discussions of solution strategies.

2) It targets mathematics students wishing to learn applications of mathematics in the life sciences and life sciences students wanting to improve their quantitative skill, which we define as the ability to look at an unfamiliar problem, analyze it to determine the necessary data, select appropriate mathematical tools, and draw conclusions on the adequacy of the results. As mentioned above, many significant research projects now require cross-disciplinary collaboration and our attempt is to replicate this experience in the classroom.

3) It reinforces students' mathematical background by exposing them to current ideas and presenting familiar mathematical topics from a novel point of view. Often after taking a course in mathematics, students never have to use the material again except in subsequent mathematics courses. This can convey the false impression that mathematics is present in the undergraduate curriculum and requirements solely as an abstract logical and algebraic exercise. We counter this with a focus on creative applications.

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[3] It might be worth mentioning that faculty who teach these courses may also, at times, feel unprepared to engage in a substantive debate on the relevant mathematical content.

4) It enables students to think in terms of mathematical models and motivates them to develop and apply modeling skills. We hope that this course will be just the first step—an introduction—to an exploration of applied mathematics that our students will undertake. We anticipate that some of the students who take the course will choose to work on more complex projects as part of SBC's summer research program or apply their expertise in different fields of the life sciences and in other courses.

Course structure

The course has three fifty-minute lectures and one three-hour lab period per week. It is divided into five relatively independent biology and physiology topics: Population Studies, Genetics, Epidemiology, Endocrinology and Biological Clocks. Each topic consists of smaller projects and takes from two to four weeks of class time to complete. For each topic, students are divided into research teams of (ideally) two biology and two mathematics students. Group work is required to develop collaborative analytical and evaluative skills. No solution strategies are provided for the projects and students are asked to discuss possible approaches and find appropriate mathematical techniques.

During lectures, in addition to introducing or reviewing information in biology and mathematics, we emphasize on development of quantitative skills: how to ask the right questions, how to answer them, and how to use the conclusions to reevaluate the situation. During the laboratory sessions, through collaborative efforts and group discussions, students propose solution strategies, identify solutions coming from them, and compare outcomes, if more than one solution strategy appears appropriate.

As our students gain proficiency, the course projects become more detailed, more realistic, and closer to actual research projects. The mathematics describing the models does not become more difficult by design, but becomes more diverse as the students discover what is needed to describe the biological process. Student grades are based on homework, laboratory assignments, and two midterm and one final course projects.

Course topics

The content of the five biological topics listed above is

- *Population Studies*: Continuous and discrete single population growth models; unlimited growth; doubling times; Verhulst's logistic growth model; chaos; continuous models governing the sizes of interacting populations; predator-prey, competition, and symbiotic models.
- *Genetics*: Dynamics of gene frequencies in a closed population; Hardy-Weinberg equilibrium; disappearance of recessive genes; analysis of continuous traits and polygenic inheritance; Gaussian distribution of polygenic traits.
- *Epidemiology*: Epidemic models in a closed system: the SIS model; the SIR model and SIR with intermediate groups; threshold of an epidemic; epidemic models with delay.
- *Endocrinology*: Pulsatile nature of hormone release, peaks in hormone time series; applications to treatment of infertility; modeling of hormone feedback networks; endocrine oscillators and the effect on growth patterns in rats; assessing the risk of severe hypoglycemia in diabetes from self monitoring blood glucose data.
- *Biological Rhythms*: Assessing the risk of neonatal sepsis from EKG data; circadian rhythms; rhythm analyses of confounded time series; microarrays and biological clocks.

Course Materials

More detailed descriptions of these topics appears in our textbook *An Invitation to Biomathematics* and the accompanying *Laboratory Manual of Biomathematics* (Robeva et al, 2008a, 2008b). The textbook provides in-depth coverage and we do not cover all of its sections in our course. More details about the course, textbook, and our entire initiative are available on our website www.biomath.sbc.edu.

Discussion

Our biggest challenge was to create an interdisciplinary curriculum that was not limited to a collection of mathematical topics with biological applications or biology topics that use mathematical methods. Biomathematics is a discipline whose methods are not a mere mix of mathematics and biology. Our course and textbook provide an interdisciplinary approach to those methods.

We considered the first offering of the course in 2002 experimental and offered it as an honors course in an attempt to attract only strong students. The results were encouraging and, since then, we have offered the course three times as a cross-listed course that students can take for mathematics or biology credit.

Although the course is accessible to sophomores, the presence of juniors and seniors enhances the learning process. We found that a form of student mentoring emerges with juniors and seniors acting as liaisons between the sophomores and the teaching faculty. This was an unexpected benefit that we have come to rely upon. It is like the peer-mentoring practices common in actual interdisciplinary research.

We found that sophomores are able to absorb ideas and information from high-level biomathematics research and, by the end of the course, provide solutions to open-ended biomathematical problems. We also found that interdisciplinary team-teaching is different from teaching traditional courses and that teamwork created synergy between students with different backgrounds. The course structure, topic selection, and pedagogy were praised in the student evaluations. All students stressed their increased understanding of the importance of interdisciplinary collaboration. One student described the course as “the learning environment of the future” and another one said the course was “the most intellectual class offered at SBC.” We attribute our success to the small class size, the informality of the seminar-like format, and the use of software that allows us to keep focused on the big picture while leaving technical details aside.

As part of our broader initiative to integrate the mathematics and biology curricula, we have used some of the course topics as stand-alone modules in conventional mathematics and biology courses including Calculus II, Statistics, Ordinary Differential Equations, Mathematical Modeling, Genetics, and Microbiology. This may be an alternative way to use our approach and materials at institutions where an integrated course in biomathematics may not be feasible.

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