Mathematical Biology and Computational Algebra at the Sophomore Level

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Name of the Institution: Chicago State University	
Size	about 7500 students.
Institution Type	Chicago State University is a public, comprehensive university that provides access to higher education for students of diverse backgrounds and educational needs.
Student Demographic	Mathematics majors with prerequisites of linear algebra, probability and statistics course, and two specific freshman biology courses. Biology majors with prerequisites of a sophomore botany course and a sophomore zoology course.
Department Structure	Mathematics and Biology are two MS degree granting departments in the College of Arts and Sciences.

1 Abstract

Biology provides a rich source of problems that can be solved by non-infinitesimal algebraic techniques. This article discusses the content of an interdisciplinary research-oriented sophomore course in computational algebra and biology created at Chicago State University. The course is part of a an attempt to teach abstract algebra on a foundation of computational algebra done at the sophomore level.

2 Course Structure

Weeks per term: 16 weeks.

Classes per week: Two 100 minutes sessions.

Labs per week: No specific time set aside for labs; computer or experiment assignments done as needed.

Average class size: Three in the pilot; 15-20 can be accommodated.

Faculty/dept per class, TAs: Team-taught by one mathematics instructor and one biology instructor. No TAs.

Next course: Not yet approved. It will be a course in computational algebra.

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3 Chicago State University, CSU

Chicago State University has consistently led Illinois public universities in the conferring of baccalaureate and masters degrees to African American students. CSU graduates more African Americans with a masters degree in mathematics than all other Illinois private and public colleges and universities combined, and is a national leader in graduating African Americans with a masters' degree in mathematics.

3.1 Department of Mathematics and Computer Science

The department has nineteen faculty members whose active research areas include mathematical biology, geometric algebras, logic, network security, software engineering, programming languages, and analysis (real, complex, and numerical, ODE, PDE). It has shifted focus from being service oriented to offering a research oriented curriculum that prepares students to work in academic and industrial settings.

The following table summarizes the distribution of students in the undergraduate majors and graduate programs.

MAJOR	02-03	03-04	04-05	05-06	06-07	07-08	08-09
Math UG	73	79	76	70	53	35	46
Math GR	27	30	21	25	22	22	19
Computer science UG	204	184	134	119	94	87	82
Computer science GR	4	9	15	14	12	13	8

4 Background and Motivation

This paper describes a part of on-going attempts to develop a computational algebra curriculum at the sophomore level that will lay a foundation for abstract algebra done at the junior/senior level. Genetics provides a source of real-world problems for computational algebra. At the MAA MathFest 07 the authors made a presentation on the project.

In private industry, mathematicians are most often part of interdisciplinary teams working closely with biologists, physicists, economists, etc., to solve complex problems (U.S. Department of Labor, 2010). Where in the undergraduate curriculum do students get an experience that will help them to see how mathematicians work, and to decide if that is right for them? Can we get it to them early enough so that it can be useful to their career planning?

In particular, mathematics curricula should prepare majors to have the skills needed to work in teams and bring their mathematics expertise to bear upon interdisciplinary problems. Thus mathematics majors should gain experience in solving a significant real-world problem and in working with individuals having other majors.

We designed *Introduction to Research in Mathematical Biology I*, Math 2180, to provide such an experience. In particular, it provides:

- An early opportunity for students to work in interdisciplinary teams on a real-world research problem.
- An opportunity for students to be the experts in their content areas.
- An opportunity to use relatively elementary mathematics to work on research problems.
- An opportunity to work on a research problem that could continue throughout their undergraduate careers.

5 Description of the Course

The course was open to both mathematics and biology majors as an elective. It was team-taught by a mathematics and biology professor, and was offered concurrently by both departments to ensure a mix of both majors.

A key idea was to build teams of students with differing abilities. Biology and mathematics majors brought different experience and expertise to the class, and then worked together to develop and analyze mathematical models. Both mathematics and biology majors needed to know enough about the other area so that they could combine their ideas. For that reason, the prerequisites for biology majors and mathematics majors were different. A mathematics major needed to have a first course in linear algebra, a probability and statistics course, and two freshman biology courses: A Survey of the Plant Kingdom, and A Survey of the Animal Kingdom. A biology major needed to have College Algebra, a sophomore botany course (Biology of Algae, Plants and Fungi), and a sophomore zoology course (Biology of Animals).

The initial class (Spring 2007) had only four students (one biology major and three mathematics majors; one of the math majors was also a biology minor). One of the math students attended the summer 2007 mathematical biology workshop at the Mathematical Biology Institute, The Ohio State University. The low number was due to two reasons: (a) the advisors were not well-aware of the course, (b) the course is an elective. The department has applied to the college to classify the course as one that satisfies the interdisciplinary requirement beginning Fall 11.

The course met twice a week in a computer lab or a biology lab. The beginning of the course saw a heavier emphasis on mathematics instruction. In particular, the students used Excel spreadsheets to study difference equations, Excel add-ins (PopTools) for analyzing population demography data, and MATLAB[1] in matrix modeling of population dynamics. Students learned structured population models, transition matrices, methods for finding eigenvalues/eigenvectors and their interpretations in terms of stability. In other words, most of Chapter 4 of Caswell's *Matrix Population Models* (2006) with introductory material from Allman and Rhodes' *Mathematical Models in Biology* (2004) were covered. Students also learned to compute powers of matrices using eigenvalues and to approximate eigenvalues using the power method.

[1]MATLAB was acquired with an internal grant from the Research Development Office of the College of Arts and Sciences.

Once the students have enough math background, they move on to more biological aspects of the course. In particular, they study models of molecular evolution and phylogenetics, using Excel to explore evolution of an invasive plant over time and space. This is basically Chapters 4 and 5 of the Allman and Roads text, omitting the math background material in the text. Also discussed were Jukes-Cantor and Kimura models, corresponding phylogenetic distances, and log-det distance. The students also constructed phylogenetic trees and learn Neighbor Joining and Maximum Parsimony methods.

As the course was envisioned, a major emphasis in the course was placed upon a math major and a biology major working in pairs (or small groups) on a research project. The small initial class size did not permit the full realization of this goal as envisioned in pairing students. Students collected population demography data using plants in the university's Research and Teaching Prairie Garden and used long-term population demography data of endangered plants collected by the biology professor. The students then used matrix population models learned in the course to project population persistence.

The course is offered at the sophomore level in order for students to do a long-term research project. The math undergraduate student joined the graduate program in mathematics and took two classes in genetics. As an extension of her work in population models, she is now finishing her masters' research project to model endangered plant populations incorporating inbreeding with data gathered from Illinois and Indiana state parks.

The course was again offered in Fall 10 but only for two senior graduating mathematics majors needing an elective. One of them became interested in mathematical biology and now at the process of applying for a NSF Graduate Fellowship Program (GRFP). The student's proposed research plan for the GRFP is based on Chicago State's aquaponics program, and is a significantly more theoretical extension of her research done for the course. She is gathering data on the length relative positions, velocities of fish, concentration of nitrites in fish effluent before and after being absorbed by the plants, and the total weight of supported plants. The data will be used for computations on the angle between velocities of different fish, minimal distance between fish, and plant absorption rates of nitrites, and estimating the total weight of the fish. This research will span at least two semesters.

6 Conclusion

Promoting the use of relatively elementary non-infinitesimal modeling techniques and reliance on technology is a hallmark of this project. It demonstrates that even at the proposed level, mathematics and biology students can interact significantly. Students inclined toward applied mathematics can meaningfully experience mathematics in a biological setting.

References

Allman, E. S. and J. A. Rhodes, 2004: *Mathematical Models in Biology.* Cambridge University Press, 370 pp.

Caswell, H., 2006: *Matrix Population Models.* Updated 2nd ed. Sinauer Associates, Inc., 722 pp.

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