Modeling Nature and the Nature of Modeling—an Integrative Modeling Approach

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Name of Institution University of Minnesota Twin Cities	
Size	About 52,000 students
Institution Type	Large land grant university with graduate and professional programs
Student Demographics	Undergraduate and graduate students in biological sciences programs
Department Structure	The Department of Ecology, Evolution, and Behavior is in the College of Biological Sciences

Abstract

Modeling Nature and the Nature of Modeling is a sophomore level course that covers aspects of linear algebra, differential equations, difference equations, and stochastic processes that are relevant to biology majors. The course emphasizes active learning, in-class computer experiments, and integrates data into modeling. While the course is mathematically rigorous, it emphasizes experimental exploration of concepts before their rigorous treatment.

Course Structure

- Weeks per term: 15
- Classes per week/type/length: two 75-minute integrated lecture/computer lab each week
- Labs per week/length: integrated with lecture
- Average class size: 15-20 students in one section
- Enrollment requirements: some familiarity with calculus (one semester)
- Faculty/dept per class, TAs: Taught by one instructor
- Next course: This course gives students an introduction to modeling that can serve as the basis for graduate level research in the biological sciences or prepare students for interdisciplinary mathematics courses with a biology focus that are available in the School of Mathematics
- Website: http://bioquest.org/numberscount/courses/bshs-calculus-2/

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Introduction

The course Modeling Nature and the Nature of Modeling was offered at the University of Minnesota Twin Cities through the Department of Ecology, Evolution, and Behavior between 2003 and 2007. It is now being redesigned for a degree program in the Health Sciences at the new University of Minnesota campus in Rochester, Minnesota. The University of Minnesota is a large public research university with over 65,000 students system-wide. In Fall 2012, the Twin Cities campus had about 30,400 undergraduate students, 13,100 graduate students, and 3,800 students enrolled in professional programs. The Twin Cities has a separate College of Biological Sciences (CBS). CBS's total enrollment of undergraduate students is about 1,700. Most of the undergraduate students choose biology as their major. Other majors are biochemistry; ecology, evolution & behavior; genetics, cell biology & development; microbiology; neuroscience; and plant biology. The campus in Rochester was established December 2006 to provide graduate and undergraduate education in health sciences and biotechnology. It started to admit undergraduate students into a Bachelor of Science in the Health Sciences in Fall 2009.

The course Modeling Nature and the Nature of Modeling was offered for the first time in Fall 2003 and subsequently every fall semester to undergraduate and graduate students on the Twin Cities campus until Fall 2007. A version of the course was offered on the Rochester campus starting Fall 2010. Originally, the course was designed to meet the need for further mathematical training after biology students completed a year of calculus to better prepare them for the increasingly quantitative nature of biology. The course is currently being redesigned to serve as the second semester of calculus for students in the new degree program in the Health Sciences on the Rochester campus.

Undergraduate students can take the course after completing a semester of calculus. There is no requirement for an upper division biology course. Graduate students who enrolled in the course on the Twin Cities campus came from various graduate programs in the life sciences and often took their last mathematics course years prior to enrolling in the course. While their understanding of biology and their exposure to primary literature is greater than that of the undergraduate students in the course, they remember little of the calculus they learned as undergraduate students.

When the course was originally designed, all the CBS biology majors were required to take a full year of calculus. Biology majors at the University of Minnesota, Twin Cities, have a choice between regular Calculus or Calculus with Biological Emphasis. The regular calculus course is based on Stewart (2007). Calculus with Biological Emphasis is based on Neuhauser (2004). While the regular calculus course does not cover linear algebra, multivariable calculus, and differential equations in the first year, Calculus with Biological Emphasis introduces students to these topics in the second semester. Since these two first-year calculus courses are taught in large lectures with weekly recitations and no computer labs, the experience of biology majors was limited to what could be done using paper and pencil and graphing calculators. This traditional approach, even in the calculus course for biology majors, makes it difficult for students to experience the rich behavior of these models. The course described here can replace a standard or biology-emphasis second semester of calculus. Materials for the course are available for download at http://bioquest.org/numberscount/courses/bshs-calculus-2/. In addition, I used my Calculus for Biology and Medicine books as supporting material.

The design for the Modeling of Nature and Nature of Modeling course was based on the need for students to learn some of the theory behind the major modeling approaches in biology and to be able to experiment with models, much as biologists explore the natural world, and to be able to interpret in biological terms the equations that describe the models. Connecting mathematical concepts to examples from the natural world is a powerful way to enhance students' conceptual understanding and their ability to read the equations that describe models. Specifically, connecting mathematical symbols with concrete examples helps students to make sense out of the often overwhelming mathematical expressions in journal articles. With funding from the Howard Hughes Medical Institute, the course is being redesigned to provide a deeper link to data and to integrate it with biology courses, including physiology and biochemistry.

Course Description

The course covers major modeling approaches in biology: difference equations, differential equations, partial differential equations, and stochastic processes. Obviously, it is impossible to provide a thorough coverage of all these topics in one semester; instead, the focus is on introducing students to them using biological examples for motivation. The goal is to make students comfortable with the mathematical description, providing them with tools for analysis, and experimental ways to explore the models. Theory is introduced when needed and primary literature is provided as background reading.

The course starts with a data set of exponential growth of monk parakeets in the U.S. (Van Bael and Pruett-Jones 1996) and asks students to build a model to describe it. Students are given the data in a spreadsheet and asked to plot it and transform axes to determine the kind of growth. The data has a stochastic component, which students learn to separate from the general trend. The data set introduces students to the simplest growth model, motivates the study of deterministic, discrete-time models and the need to understand how to model stochasticity. These concepts are explored subsequently, using spreadsheet exercises and introducing the students to a more powerful and sophisticated modeling software (Matlab). Students learn to simulate stochastic processes in both a spreadsheet and Matlab. They also learn how to mathematically describe and analyze deterministic growth and growth in a random environment, including how pseudo-random numbers are generated and the importance of the geometric mean of the growth parameter.

The first two-thirds of the course moves quickly through difference and differential equations, emphasizing in-class exploration of the models on the computer, and in-class short group projects where students can practice the more theoretical aspects of the analysis. Difference and differential equations are introduced using population models, and students learn about equilibria and stability. After covering some of the concepts of linear algebra that are needed to analyze systems of difference or differential equations (primarily eigenvectors and eigenvalues), students learn how to solve systems of linear difference and differential equations and to analyze the long-term behavior of systems of non-linear difference and differential equations by linearizing about the equilibrium. The biology behind population models is easy to understand and allows students with different levels of biological understanding to become familiar with modeling biological phenomena without being overwhelmed by both the mathematics and the biology.

A brief introduction to reaction-diffusion and advection-dispersion equations gives students a taste of more complex models. Because of their mathematical complexity, the focus is on understanding the physical meaning of the terms that appear in the equations and to show some of the complex behavior they may exhibit, including pattern formation.

For most students, this is the first time that they experience complex behavior exhibited by mathematical models, and they are surprised by the rich behavior. Whether in a single discrete time equation or in systems of partial differential equations, they begin to appreciate the value of mathematical modeling in aiding our understanding of complex behavior. Students are also introduced to bifurcation analysis. This is rarely taught to biologists but is a very powerful way to analyze the behavior of a mathematical model.

Once students feel comfortable with the various modeling approaches, more complex applications are studied. Examples include biochemical networks, genetic switches, and models of the cell cycle. The purpose of studying these examples is to give students the opportunity to read and understand theoretical papers, using the knowledge they gained in the first two-thirds of the course.

Because of the diverse backgrounds of the students and their different levels of mathematical preparedness, exams in the course are take-home exams where students are allowed to consult books, course notes, and the web. They can seek help from the instructor, and depending on the amount of help they receive, they might have to sacrifice some points. Students are encouraged to view exams as small research projects and are expected to explain their answers as if they were writing a technical report. For instance, one exam question gave students a Leslie matrix for a population of ruffed grouse and asked them to examine the viability of the population (the growth parameter was below the critical value for survival) and to justify management strategies that would rescue the population. Students were told that the information was limited and that they should view the data as a first step in initiating further study to develop a more comprehensive management plan. Their answer was to be given to a manager at a Department of Natural Resources and so had to include explanations that went beyond just listing the largest eigenvalue of the Leslie matrix for different scenarios.

The course also includes short writing assignments that ask students to reflect on the material or on aspects of learning mathematics.

Discussion

I have taught the course five times and continue to revise it to incorporate what I learn from teaching the materials. In its first year, the course was split into a weekly lecture followed by a computer lab. The seventy-five minute lecture proved to be too much theory at once. Students could not absorb all the theory and then apply it in the following computer lab. The course felt disconnected.

Starting in the second year, I increasingly combined lecture and lab. Lecture and lab are now completely integrated, seamlessly moving between in-class model explorations and short segments of theory as needed. The course was taught in a computer lab to groups of 10-15 students. The next iteration of this course will integrate data sets into the modules to allow students to explore model behavior using real data with the goal of providing better motivation for the various models.

A major change in the mathematics requirements in CBS on the Twin Cities campus required a redesign of the course to accommodate those who had less exposure to calculus. Most undergraduate majors in the College of Biological Sciences are now required to take only one semester of calculus. The requirement for the second semester of calculus was replaced by either a non-calculus-based statistics course or a computer science course. Fortunately, a second semester of traditional calculus with its focus on integration techniques is not needed to learn modeling based on difference and differential equations. In fact, the course will serve as the second semester of calculus in the new degree program at the University of Minnesota Rochester and will be offered again starting Fall 2010.

One semester is short for a course that covers the major modeling approaches, and we cannot expect students to be proficient modelers after one semester. The course, however, removes some of the barriers biology students with just one semester or one year of calculus face when reading scientific papers that include modeling.

Since students typically do quite well in the course (being able to use books, notes, and the internet during exams contributes to their success), the course also serves as a confidence builder. After it, students can read equations that describe mathematical models more effectively and are less likely to skip over equations in papers. They are also more willing to experiment with models and explore their behavior.

References

Neuhauser, C., 2004: *Calculus for Biology and Medicine*. 2nd edition. Prentice Hall.

Stewart, J., 2007: *Calculus: Early Transcendentals*. 6th edition. Cengage Learning.

Van Bael, S. and S. Pruett-Jones, 1996: Exponential population growth on Monk Parakeets in the United States. *Wilson Bulletin*, **108**, 584-588.