Biocalculus at Benedictine University

Timothy D. Comar[1], Department of Mathematics, Benedictine University

Name of Institution	Benedictine University
Size	3500 undergraduate students, 8900 students in total
Institution Type	mid-sized university primarily devoted to undergraduate education with some graduate and professional programs
Student Demographic	recent high school graduates and community college transfers with interest in pursuing medical careers
Department Structure	Mathematics and Biology are individual departments in the College of Science

Abstract

This paper describes the biocalculus course sequence at Benedictine University. Benedictine University offers a two-semester biocalculus course sequence that integrates calculus-based mathematics, biological models, and computational software. In addition to the biocalculus courses, all first semester calculus students at Benedictine University take a computer laboratory course, which includes several biological applications. The paper concludes with a discussion of the success of the courses and of the challenges still facing them. This article is unique in that it describes a rigorous, introductory mathematical biology course sequence that can be implemented at small to mid-sized institutions in which only a limited number of sections of calculus are offered each year.

Course Structure

- Weeks per term: 15 weeks
- Classes per week/type/length: Biocalculus I: 4 50 minute lecture periods per week; Biocalculus II: 3 50 minute lecture periods per week
- Labs per week/length: Calculus with Analytics I Lab/Biocalculus II Lab: 1 2 hour period per week
- Average class size: Biocalculus I: about 15 students in each of two sections; Biocalculus II/Biocalculus II Lab: 5-10 students in one section; Calculus with Analytics Lab: up to 20 students
- Enrollment requirements: Biocalculus I: placement exam; Biocalculus II: a grade of C in Biocalculus I or Calculus with Analytics I
- Faculty/dept per class, TAs: One mathematics structure per class; same instructor for Biocalculus II and Biocalculus II Lab
- Next course: Biocalculus II/Biocalculus II Lab follows Biocalculus I. Students may take Calculus with Analytics III group or Mathematics Research after Biocalculus II.

Introduction

Benedictine University is private, primarily undergraduate institution located in the western suburbs of Chicago. Of the 3500 students enrolled as traditional undergraduates, over 500 are majoring in the biological and health sciences. Benedictine University has a reputation for involving undergraduate science and mathematics majors in research and regularly sends graduates to professional and graduate programs. The biocalculus course sequence at Benedictine was developed in response to the *Bio 2010* report (National Research Council 2003) to increase the mathematical and quantitative training for future researchers in the biological sciences and to provide a relevant calculus experience for biology students. There is also a recent push at Benedictine University for students to engage in interdisciplinary research involving mathematics

[1]tcomar@ben.edu

and biology. The course sequence is now the required mathematics sequence for students in the research oriented Biochemistry/Molecular Biology (BMB) major, which was recently revamped in accordance with *Bio 2010* recommendations.

The course design was affected by the needs and nature of the student body and the Department of Mathematics' ability to staff calculus courses and laboratories. We have considered students who will take only a single semester of calculus, students who need a year of calculus-level mathematics, and students who intend to take more mathematics courses. When the courses were designed, trigonometry was the required mathematics course for biology and health science majors. We also have students who have been admitted into a dual acceptance program with a local pharmacy school. These students spend only two years at Benedictine University and have room for only one semester of calculus. Because of students who will only take one semester of calculus, it is important that the first semester biocalculus course include biological models and biologically-oriented computer laboratory projects.

We realize that majors in the biological sciences may not have the credit hours available to take additional mathematics courses beyond a year of calculus as undergraduates. We have chosen mathematical topics that enable students to develop a foundation in calculus and to be able to use mathematical and computational techniques to analyze biological mathematical models represented by systems of difference equations or differential equations by the end of the second course. On the other hand, it is necessary at our institution to keep the biocalculus sequence and the traditional calculus sequence at the same level of rigor so that students can switch tracks for their second course and so that all students would be prepared to succeed in a traditional third semester calculus course.

Another significant and unique aspect of first semester calculus at Benedictine University is that all students in the traditional first semester calculus course and the first semester biocalculus course must enroll in a common laboratory course, Calculus with Analytics I Laboratory (hereafter, Lab I). The course schedule is flexible so that it can be used for mixed classes of students from both versions of first-semester calculus. The course also lets us assess and compare performance between students in Biocalculus I and the traditional Calculus with Analytics I (hereafter, Calculus I).

Biocalculus I was first offered in Fall 2003, and Biocalculus II was first taught in Spring 2005. Enrollment levels initially ranged from ten to sixteen students in Biocalculus I and from five to six students in Biocalculus II. Currently, there are two sections of Biocalculus I offered each fall with average enrollment of about fifteen students, and the spring section of Biocalculus II has enrollment of about ten students. For comparison, each fall semester there are two sections of Calculus I with class size of about thirty students. Biocalculus I is open to all students who place into first semester calculus and is recommended for those majoring in the BMB program, biology, or the health sciences. Lab I was first offered in Spring 2005, with its first mixed class of Biocalculus I and Calculus I students in Fall 2005. Multiple sections of Lab I with at most twenty students per section have been scheduled to accommodate the students in both calculus courses. We also offer two sections of a year-long integrated calculus and precalculus course sequence, Introduction to Calculus and Applications of Calculus, each year, with approximately sixty students per year divided into two sections. Each term, we also offer one section each of traditional second and third semester calculus courses. The biocalculus course sequence and accompanying laboratory courses are:

Table 1: Biocalculus and Laboratory Courses

Course	**Hours**
Biocalculus I	4 credit hours
Calculus with Analytics I Laboratory	1 credit hour, 2 contact hours
Biocalculus II	3 credit hours
Biocalculus II Laboratory	1 credit hour, 2 contact hours

The reader may find detailed syllabi and laboratory projects at the author's website:

http://www1.ben.edu/faculty/tcomar/index.htm

Description of the Courses

The philosophy behind the pedagogy of our biocalculus courses is the balance of mathematical content, rigor, and reasoning, biologically oriented examples and modeling applications driven by the use of real data where possible, and the use of computational tools to investigate biological and mathematical phenomena. Our presentation is infused with the Rule of Four popularized by the calculus reform movement. (The Rule of Four is the presentation of mathematical content symbolically, numerically, geometrically, and verbally.) Emphasis on the multiple representations of quantitative information is important for science students because they are more likely to work with data presented numerically or graphically in their science courses. Those who pursue avenues in which symbolic mathematical representations are relevant will have the skills to translate between verbal descriptions of phenomena and their expressions in mathematical notation.

Biocalculus I and Calculus with Analytics I Laboratory

We now describe in greater detail the Biocalculus I course and its connection with the common laboratory course. The syllabus, listed in Table 2 for Biocalculus I, was developed to meet the needs of our students and to accommodate the institutional considerations described above. At first glance, the course may seem not much different from a traditional first semester calculus course. This is significant because a similar syllabus can be implemented for a course to run concurrently with traditional calculus courses (as is done at Benedictine University) or can replace the traditional first semester calculus course without impacting the second semester of traditional calculus (as is done at Macalaster College (Bressoud 2004)). Table 2 includes some topics generally not found in the traditional first semester of calculus. An obvious question is how to adjust the content of the traditional material to accommodate the additions. Part of our solution was to move several topics, listed in Table 3, to the concurrent lab course. Other topics need to be streamlined. For example, limits of sequences are addressed thoroughly, including a discussion of formal definitions, before discussing limits of functions of real variables. When introducing the latter, we point out similarities between the two collections of limit laws and have already established a context in which we can efficiently present the formal ϵ-δ definition of a limit. Related rates are present, de-emphasized. Less time is spent on optimization application problems because other applications occur throughout the course.

Table 2: Topics for Biocalculus I

Review of functions
Nonlinear scales and allometry
Introduction to probability
First order difference equations, sequences, and population models
Limits and continuity of functions of a real variable
Differentiation
Applications of differentiation (including stability of difference equations)
The definite integral and Riemann sums
Antidifferentiation
The fundamental theorem of calculus
Basic applications of integration
Integration by substitution

The most important addition to the syllabus in Biocalculus I is difference equations. There are several advantages to studying these in a first semester calculus course. From the biological perspective, students can begin to understand some basic discrete population models without significant mathematical or biological background. Students learn how to represent discrete population models symbolically, how to implement them in Excel and Maple, and how to study their dynamics. From the mathematical perspective, there

Table 3: Calculus Topics in Calculus with Analytics I Laboratory

Conic sections
Parameter curves
Curve sketching
Numerical integration

are significant pedagogical benefits. If difference equations are introduced before limits of functions of a real variable, students can learn about sequences early. Convergence of sequences follows naturally from how students observe limits numerically and geometrically: they can observe to what number a sequence of numbers converges by looking at a table of values or a graph. Formal definitions of limits of sequences can then be introduced. Locally stable equilibria can be interpreted as values to which a sequence can converge. The limit of a function of a real variable can then be built on the limit in the discrete case. To help motivate the definition of the derivative, we can think of differences of successive terms as difference quotients having a unit difference between successive input values. Such differences provide a discrete approximation to the derivative. As the difference between successive input values decreases to zero, the derivative results. The criterion for local stability of a first order difference equation provides a nice application of differentiation, as it follows from using a tangent line approximation to linearize the difference equation. The stability result is a nice analog to the stability result for first order differential equations.

One of the themes discussed at the Biology CRAFTY Curriculum Foundations Project was that “Creating and analyzing computer simulations of biological systems provides a link between biological understanding and mathematical theory” (Dilts and Salem 2004). Additionally, the *Bio 2010* report asserts the importance for biologists of being able to use computers as tools: “Computer use is a fact of life of all modern life scientists. Exposure during the early years of their undergraduate careers will help life science students use current computer methods and learn how to exploit emerging computer technologies as they arise” (National Research Council 2003). These needs are met in the common laboratory course, Lab I (Comar and Townsley 2006) and through additional computational activities and projects in the Biocalculus I course.

The laboratory course, Lab I, is required of all students enrolled in either Calculus I or Biocalculus I. It helps to keep both first semester calculus courses at the same level and pace while providing time for content not covered in the lecture course and to explore additional applications, including biological applications. Students are introduced to the computer algebra system (CAS), Maple, as a calculating and graphing tool to analyze problems. (Maple is used throughout the calculus sequence at Benedictine University.) An important feature of the laboratory course is its collaborative nature. Students from the first semester calculus courses work together, enabling them to share their expertise to solve problems together, corresponding to the collaborative nature of academic research environments and industry. As much scientific research is done at the intersection of disciplines and as biological research is becoming increasingly computational and quantitative, it is important that science and mathematics students learn to communicate with each other. The lab experience offers biology majors and mathematics majors an opportunity to work together and develop a common language. Currently, three of the weekly projects in the course investigate biological models (see Table 4).

One of the goals of the biocalculus course sequence is to examine a wide array of biological problems using different computational platforms. The platform choice for a particular model depends on the ease of implemention and the applicability to future course or research work. Maple and Excel are used in Biocalculus I. The latest version of Maple is user-friendly and is appropriate for calculus courses. Moreover, activities can be developed in Maple so that the students are not burdened with programming and notational complexities. Maple is one of the leading commercially available computer algebra systems; those who pursue research are likely to encounter it or a similar computer algebra system for symbolic computation. We frequently use Excel for difference equation models because it is easy and convenient to implement recursion. Excel is ubiquitous and popular among biologists, so the students are likely to encounter it after their calculus course experience.

Table 4: Biologically-oriented Projects in Calculus with Analytics I Laboratory

Data fitting, nonlinear scales, allometric relationship between tree height and diameter
Stability of first order difference equations and cobwebbing using a discrete population model
Introduction to differential equations: slope fields, phase line diagrams, and Euler's method using the logistic model

Biocalculus I students work on several computational activities throughout the term. Each explores at least one biological model and analyzes the effects of varying model parameters. The first project uses Maple to study a drug dosage model that, from the mathematical perspective, enables the students to analyze discontinuous functions with parameters. Another activity is a curve fitting problem in which the students fit a logistic curve to data measuring the length of a growing bean leaf over time. Students learn to implement discrete population models using difference equations in Excel and Maple. Both allow students to plot time series and cobweb diagrams and see the effects of varying parameters. Maple is nice for showing animated diagrams that step through the cobweb process one stage at a time and for creating bifurcation diagrams. Another activity analyzes a simple difference equation model for natural selection and in Excel.

Although the computer laboratory activities provide deeper investigation of biological models and introduce students to useful software programs, they take up a significant amount of class time, and more must be allotted to discussion. Incorporation of these activities promotes breadth over depth; diminishing or eliminating less crucial topics is necessary to allow for appropriate treatment of these time-consuming activities. The computer activities are integrated throughout the course during class time and through assignments. The course includes six or seven group homework assignments, two in-class exams, and a final exam, each of which has computer components.

Biocalculus II and Biocalculus II Laboratory

Unlike the first semester course, Biocalculus II deviates substantially from a traditional second semester calculus course. It is a hybrid of a second semester calculus course and an introductory course in mathematical biology. Its goal is to use calculus, related mathematical tools, and computational software to develop an understanding of biological mathematical models using difference equations, differential equations, matrix models, systems of difference equations, systems of differential equations, and the basics of probability. The course asks students to use mathematical and computational tools to investigate complex biological systems as a scientist. This approach is different from that of traditional mathematics courses, where the focus is simply on the mathematics. Topics have been chosen to achieve two distinct objectives (see Table 5. One objective reflects our understanding that students in this course may not be able to take another mathematics course. We present enough mathematical topics and provide experience analyzing biological models so that students are equipped to succeed in quantitative science courses and research experiences. The second objective is to ensure that students who complete the second semester of biocalculus will have developed sufficient mathematical maturity and background to succeed in a traditional third semester calculus course. Several former Biocalculus II students have successfully continued with Calculus III and additional mathematics courses.

Even though the Biocalculus II and the Biocalculus II Lab courses are separate courses, they are taught by the same instructor and comprise an integrated experience. Like the common lab course for first semester calculus students, the Biocalculus II Lab course is a one-credit hour course that meets for a two-hour session once a week. This provides five contact hours each week, as compared to the four hour weekly lecture time in the traditional second semester calculus course. We schedule projects throughout the term. They are biological and require work outside of class. The list of project titles is in Table 6. To help prepare students

Table 5: Biocalculus II Topics

Review of Biocalculus I and integration by substitution
Integration techniques: integration by parts, simple partial fractions, use of tables and computer algebra systems (Optional)
Indeterminate forms, L'Hôpital's rule, and improper integrals
Continuous probability distribution functions
Application of integration to volume calculations
First order differential equations with stability and bifurcations
Analytic geometry: vectors and equations of lines and planes
Matrix algebra: matrix properties and operations, including the determinant, the trace, eigenvalues and eigenvectors
functions of several variables
Limits of functions of several variables
Partial derivatives
Differentiability of real-valued and vector-valued functions of several variables, linearization
Nonlinear systems of difference euations
Linear and nonlinear systems of differential equations

Table 6: Biocalculus II Lab Projects

Taylor polynomial approximations
Improper integrals and life tables
Stability of single ordinary differential equations, bifurcations, logistic model with harvesting
The Leslie matrix model for age-structured populations
Systems of nonlinear difference equations: host-parasitoid models
Systems of linear differential equations
Systems of nonlinear differential equations I: competition and predator prey models
Systems of nonlinear differential equations II: the Fitzhugh-Nugamo model

for research activities, an extended project has become the capstone of the course. The students read a research paper in mathematical biology, implement the model using an appropriate computation platform, and prepare written and oral reports about the model and their implementation. The workload for Biocalculus II also includes four group homework assignments, two in-class exams, a final exam in the lecture course, and a group final exam in the laboratory course.

Discussion

Successes

The biocalculus courses at Benedictine University have proven to be a success. First semester biocalculus and traditional calculus students perform comparably on calculus skills and in the laboratory course (Comar 2008); hence the two courses are taught at the same level of rigor and provide the same calculus skills. Other success are the integration of biological content into the common laboratory course and the success of students in succeeding courses and research experiences. Most importantly, the courses are now established in the curriculum.

We have two assessments that measure the performance of students in both first semester calculus courses, which enable us to ensure that students who successfully complete either course have demonstrated proficiency in a common core of calculus concepts and skills and will be able to succeed in either second semester calculus course. The first assessment tool is a set of common final exam problems. The second is the common laboratory course, which provides a unique opportunity to compare first semester calculus students. Assessments in the laboratory course included the weekly laboratory activities, a written component of the final exam covering the calculus content specific to the lab course, and a collaborative component of the final exam testing modeling, problem solving, and use of a computer algebra system. Our data confirm comparable success rates (Comar 2008). Two conclusions are that students can switch between traditional calculus and biocalculus tracks and that students who complete Biocalculus I have the same fundamental calculus skills as those in the traditional Calculus with Analytics I course. This supports the biocalculus course as an alternative to the traditional first semester calculus course.

Students who have taken both biocalculus courses have had successful research experiences and have worked on projects at the intersection of mathematics and biology. These are key goals of the biocalculus courses, so the biocalculus course sequence is working. Even though the biocalculus sequence may be a terminal experience for most students, several have taken additional mathematics courses. Ten of the twenty-nine students who have completed Biocalculus II have registered for Calculus III. Eight of these earned grades of C or better, 1 student earned a D, and 1 student withdrew early in the term for personal reasons. Three of these eight students have become mathematics minors and three have added mathematics as a major. Five of the twenty-nine have participated in our Summer Science Research Program. Several others have pursued research activities during the academic year. (Research is required of all BMB majors.) One successful Biocalculus II student completed an integrated biology and mathematics project for the major research project in Benedictine University's Scholar's Program, which is the honors program at the university. Another able student has been hired to work with mathematics and biology faculty as a research assistant on a biocalculus textbook project. Several other biocalculus students who completed only Biocalculus I have participated in the Summer Science Research Program and have completed research projects as undergraduates. All students who have taken Calculus II following Biocalculus I have been successful, including two who became mathematics majors and one who completed a minor in mathematics.

The most important success of the biocalculus courses is that the courses have been institutionalized. Before the biocalculus courses were created, trigonometry was the only mathematics requirement for biology majors. Now, one semester of calculus is required for Bachelor's of Science biology majors and two semesters are required for the BMB majors. Biocalculus I is the recommended course for the first group and Biocalculus II with the lab is required for the second. This has contributed to the increase in enrollment in the courses and has enabled the university to offer two sections of Biocalculus I each fall. Instructors other than the course designer have been able to teach the courses successfully.

Challenges

Recruitment into the biocalculus courses remains a challenge. There is a false perception that they are significantly more difficult than the traditional calculus courses. Moreover, some parents have felt more comfortable placing their freshmen children in a course called "Calculus" than "Biocalculus" because they are more familiar with the word "Calculus." Hence a significant number of biology and pre-med students still register for Calculus I. Freshman advisors are working with the Department of Mathematics to encourage students majoring in BMB, biology, and health sciences to take Biocalculus I. Biocalculus II consistently has a lower enrollment than Biocalculus I because fewer students are required to take the second calculus course than the first. Moreover, not all students who complete Biocalculus I are interested in the research orientation of course.

Acknowledgments

The author would like to thank his colleagues Drs. Lisa Townsley and Manu Kaur, who have taught the traditional calculus courses. Dr. Townsley developed the Lab I course. The author would also like to thank the National Science Foundation, which is supporting the biocalculus program at Benedictine University and College of DuPage through the NSF CCLI grant #DUE-0633232.

References

Bressoud, D.M., 2004: The changing face of calculus: first- and second-semester calculus as college courses. *Focus*, **24**, No. 8, 14-16.

Comar, T.D., 2008: Integration of Biology into Calculus Courses. *PRIMUS*, **18**, 47-70.

Comar, T.D., and L. Townsley, 2006: A common calculus laboratory course for traditional calculus and biocalculus students. *Proceedings of the 18th ICTCM*, Pearson Addison Wesley, Boston, MA.

Dilts, J., and A. Salem, 2004: Biology: CRAFTY Curriculum Foundation Project, Macalester College, November 2-5, 2000. *Curriculum Foundations Project: Voices of the Partner Disciplines*, S. Ganter and W. Barker, Eds., The Mathematical Association of America, 15-18.

Edelstein-Keshet, L., 2005: Adapting mathematics to the new biology. *Math & Bio 2010: Linking Undergraduate Disciplines*, L. A. Steen, Ed., The Mathematical Association of America, 63-73.

National Research Council, 2003: *Bio 2010: Transforming Undergraduate Education for Future Research Biologists*, National Academies Press, Washington, D.C.