1. Find the general solution to the ODE.

    [3] a. \( u' = -(\tan t)u + \sec t \).
    [3] b. \( \frac{dv}{dx} = (1 + x^2)(1 + v^2) \).
    [4] c. \( y' = \frac{2y}{x} - x^2 y^2 \).

2. Consider the first order ODE \( u' = f(t, u) \).

    [7] b. Explain the use of the direction field in graphically solving the equation. In particular, how do you know that the curves you’ve sketched are the graphs of solutions?

3. The velocity \( v = v(t) \) at time \( t \) of a particle of mass \( m \) falling downward through a viscous fluid satisfies the equation

\[
v' = g - \frac{a}{m} v^2,
\]

where \( a \) is a positive constant. Sketch the solution family. Identify the equilibrium solutions. What is the particle’s terminal velocity?

4. Consider a simple pendulum with string length \( l \) and bob mass \( m \). At time \( t \) the string makes an angle \( \theta(t) \) with the vertical. Use conservation of energy or Newton’s second law to derive the pendulum equation

\[
\theta'' + \frac{g}{l} \sin \theta = 0.
\]

5. Let \( L = D^2 - \frac{1}{t} D - 4t^2 \).

    [8] a. For \( t > 0 \), \( u_1(u) = e^{t^2} \) satisfies the homogeneous equation \( Lu = 0 \). Use reduction of order to find a solution \( u_2(t) \) that is linearly independent of \( u_1(t) \) for \( t > 0 \).
    [2] b. Solve the initial value problem

\[
\begin{align*}
Lu &= 0, \\
u(1) &= 0, \quad u'(1) = 1.
\end{align*}
\]
6. Let \( Lu = u'' + u' - 2u \).

[5] a. Find the general solution to the equation \( Lu = 0 \).

[5] b. Solve the initial value problem

\[
\begin{align*}
L u &= e^t, \\
u(0) &= 1, \quad u'(0) = 0.
\end{align*}
\]

[10] 7. Solve the initial value problem

\[
\begin{align*}
u'' + 4u &= 3, \\
u(0) &= 1, \quad u'(0) = -1.
\end{align*}
\]

[10] 8. Let \( \{u_1, u_2\} \) be a fundamental set for the second-order, linear, homogeneous equation \( Lu = 0 \). Show that the general solution is

\[
u(t) = c_1u_1(t) + c_2u_2(t).
\]

You may use the fact that \( W(u_1, u_2)(t) \neq 0 \).