Section 1.3.4, Problem 1

Since \( \ln T = -at + b \),

\[
\frac{T'}{T} = -a, \tag{1}
\]
or

\[
T' = -aT, \tag{2}
\]

which is Newton’s law of cooling with thermal loss coefficient \( a \) and ambient temperature \( T_e = 0 \). By the data given,

\[
b = \ln 22, \quad \text{and} \quad a = .5 \ln 2.75. \tag{3}
\]

Thus the time as a function of the temperature is

\[
t = \frac{b - \ln T}{a} \tag{4}
\]

\[
= \frac{\ln (22/T)}{.5 \ln 2.75}. \tag{5}
\]

The temperature reaches \( 2^\circ \) at time

\[
t = \frac{\ln 11}{.5 \ln 2.75} \approx 4.74 \text{ hours}.
\]

Notes

a. Please leave routine calculations and explanations on the scratch paper. I know that you can do basic algebra and calculus, and I’d rather not have it clutter your written work. Just give me a summary of the main steps, ideas and conclusions. You might even write less than I did in the above example. For instance, (1) and (4) are not strictly necessary to the exposition.

b. Don’t just list mathematical steps. Imbed them in sentences.