Exam 1

1. The following matrix $A$ is invertible. Find the inverse.

\[
\begin{pmatrix}
3 & 2 & 0 \\
6 & 6 & 0 \\
3 & 4 & 1 \\
\end{pmatrix}
\]

Let $b$ be the vector $[2, 0, 3]$.
How many solutions does it have the system $Ax = b$? List all the solutions.
What is the span of the columns of $A$?

2. Let $A$ be the following matrix:

\[
\begin{pmatrix}
0 & 3 & 0 & -3 & 9 \\
2 & 2 & 0 & 2 & 8 \\
0 & 2 & 2 & 0 & 10 \\
\end{pmatrix}
\]

and let $R$ be the following matrix

\[
\begin{pmatrix}
1 & 0 & 0 & 2 & 1 \\
0 & 1 & 0 & -1 & 3 \\
0 & 0 & 1 & 1 & 2 \\
\end{pmatrix}
\]

$R$ is the reduced echelon form of $A$.
Mark as true or false the following statements:

- $\text{row}(A) = \text{row}(R)$.
- $\text{col}(A) = \text{col}(R)$.
- $\text{null}(A) = \text{null}(R)$.

Find a basis for the vector space $\text{col}(A)$.
List the free variables for the system $Ax = b$ and find a basis for the vector space $\text{null}(A)$.
Find the $\text{rank}(A)$.

3. Explain why the rows of a $3 \times 5$ matrix have to be linearly dependent.

4. Let $A$ be a matrix which is not the identity and assume that $A^2 = A$. By contradiction show that $A$ is not invertible.

5. Let $u, v, w$ three linearly independent vectors. Are $u - v, v - w, u - w$ linearly independent? Explain.

6. Let $3x + 5y + 7z = 0$ be a plane. Is it the plane perpendicular to the line $x = 4t + 1, y = -t + 9, z = -t + 190$?

7. True or false: Given two matrices $A$ and $B$ is it true that $(A + B)^2 = A^2 + 2AB + B^2$?

Good Luck.