1. Let $A$ be a $3 \times 3$ matrix, which determinant is 5 and let $B$ be a $3 \times 3$, which determinant is 4.

   (a) What is the rank of $A$?
   (b) What is $\det(2A)$?
   (c) What is $\det(AB)$?
   (d) Can you compute $\det(A + B)$?

2. $\det(A) = 3$, where $A$ be the following matrix

   $\begin{pmatrix}
   a & b & c & d & e \\
   f & g & h & i & l \\
   m & n & o & p & q \\
   r & s & t & u & v \\
   0 & 1 & 2 & 1 & 0
   \end{pmatrix}$.

   (a) Can the first row be $[0, 1, 2, 1, 0]$?
   (b) Compute the determinant of the matrix

   $\begin{pmatrix}
   a & b & c & d & e \\
   f & g & h & i & l \\
   m + 3r & n + 3s & o + 3t & p + 3u & q + 3v \\
   r & s & t & u & v \\
   0 & 1 & 2 & 1 & 0
   \end{pmatrix}$.

3. Find the value of $k$ such that the following matrix is invertible

   $\begin{pmatrix}
   0 & k & 0 \\
   1 & 9 & k \\
   k & 4 & 3
   \end{pmatrix}$.

4. Let $A$ be a $4 \times 4$ matrix such that $Av = 0$ where $v$ is the column vector $[1, 2, 4, 1]^T$. What is the determinant of $A$?

5. Let $A$ be an $4 \times 4$ matrix. Let $[1, 2, 1, 0], [1, 0, 1, 0]$ be eigenvectors with eigenvalue 3, and let $[0, 0, 2, 1], [0, 0, 1, 0]$ be eigenvectors with eigenvalue 5.

   (a) Explain why $A$ is diagonalizable.
   (b) $A = P^{-1}DP$, where $P$ is an invertible matrix and $D$ is a diagonal matrix. Find $P$ and $D$.

6. Compute

   $\begin{pmatrix}
   1 & 0 & 0 \\
   0 & 2 & 9 \\
   0 & 0 & 3
   \end{pmatrix}^{10}$.

7. The polynomial $p(\lambda) = \lambda(\lambda - 3)^2(\lambda - 4)$ is the characteristic polynomial of a square matrix $A$.

   (a) what is the size of the matrix?
(b) Assume that the eigenspace corresponding to the eigenvalue \( \lambda = 4 \) has dimension 2. Explain why \( A \) is diagonalizable.

8. Let \( A \) be similar to \( B \), \((P^{-1}AP = B)\). If \( x \) is an eigenvector for \( A \) then \( P^{-1}x \) is an eigenvector for \( B \).

9. Let \( A \) be a matrix such that \( A^3 = A \). What are the possible eigenvalues for \( A \)?

10. Find \( W^\perp \) where \( W = \{(x, y, z) \mid 3x + 4y + 8z = 0, 4x - 1y + z = 0\} \).

11. Let \( W = \text{span} < w_1 = [2, 1, 3, 4], w_2 = [0, 1, 2, 1] > \) and let \( v = [1, 0, 1, 0] \).
   
   (a) is \( \text{proj}_W v = \text{proj}_{w_1} v + \text{proj}_{w_2} v \) ?
   
   (b) Compute an orthogonal basis for \( W \) with the Gram-Schmidt algorithm.
   
   (c) Compute \( \text{proj}_W v \).

12. Number 16 page 404.

13. Say which of the following is a vector space over the real numbers:
   
   (a) \( S \) is the set of real valued functions continuos over \( R \) such that \( \lim_{x \to 0} f(x) = 0 \).
   
   (b) \( S \) is the set of real valued functions continuous over \( R \) such that \( \int_1^3 f(x)dx \) does exist.
   
   (c) \( S \) is the set of polynomials \( p(x) \) such that \( p(2) = 0 \).

14. Let \( \mathcal{F} \) be the vector space of real continuos functions. Are \( 1, \cos(x), \sin(x) \in \mathcal{F} \) linearly independent?

15. Let \( \mathcal{F} \) be the vector space of real continuos functions. Are \( 1, \cos^2(x), \sin^2(x) \in \mathcal{F} \) linearly independent?