Math 101
Hour Exam II

Name______________________________________________________

1. Use algebra to solve for \( x \):
   
   a) \( x(x + 4) = 12 \)

   b) \( x^2 - 2x = 10 \)

   c) \( x^4 + x^2 - 12 = 0 \)

2. Write the following in the form \( a + bi \):

   a) \( (3 - 4i)^2 \)

   b) \( \frac{4i}{1 - 2i} \)
3. Find the equation of the line parallel to $2x - 4y = 3$ which passes through $(6, -3)$.

4. Let $f(x) = x^2 - 4$ and $g(x) = \frac{x}{2x + 6}$. Find and simplify:
   
   a) $(f - g)(-2)$

   b) $(f \circ g)(0)$

   c) $(g \circ f)(x)$

   d) The domain of $g(x)$ in interval notation

   e) The domain of $f(x)$ in interval notation
5. Simplify the difference quotient \( \frac{f(x + h) - f(x)}{h} \) when \( f(x) = 4x + 2 \).

6. Find the relative maximum and/or minimum of the function \( G(x) = 4x - 4x^3 \) and the interval(s) on which \( G(x) \) is increasing.

Maximum__________________________Minimum__________________________

G(x) in increasing on the interval(s) ________________________________

Is this function even, odd, or neither? Justify your answer using algebra!

7. Solve the quadratic equation \( x^2 - 6x - 3 = 0 \) by completing the square.

8. Find the equation of a function which resembles \( y = x^3 \) shifted right 3 units and down 7 units.
9. A stone is thrown upward from a height of 100 feet with an initial velocity of 80 ft./sec. The height of the stone t seconds after it has been thrown is given by the function \( s(t) = -16t^2 + 80t + 100 \). Find the maximum height of the stone.

10. Are the lines given by the equations \( 2x + y = 4 \) and \( 2x - 4y = 6 \) perpendicular to each other? Why or why not?

11. Sketch a graph of the function \( f(x) = \begin{cases} 4x - 6, & x \leq 3 \\ 3 - x, & x > 3 \end{cases} \)

Label at least 3 points.
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Name Answer Key

1. Use algebra to solve for $x$:
   a) $x(x + 4) = 12$
      
      $x^2 + 4x = 12$
      $x^2 + 4x - 12 = 0$

      $(x + 6)(x - 2) = 0$
      $x + 6 = 0$  $x - 2 = 0$
      $x = -6$  $x = 2$

   b) $x^2 - 2x = 10$
      
      $x^2 - 2x - 10 = 0$
      $x = \frac{2 \pm \sqrt{4 + 4(1)(10)}}{2(1)}$
      $x = \frac{2 \pm \sqrt{44}}{2}$
      $x = 1 \pm \sqrt{11}$

   c) $x^4 + x^2 - 12 = 0$
      
      $(x^2 + 4)(x^2 - 3) = 0$
      $x^2 + 4 = 0$  $x = \pm 2i$
      $x^2 = -4$
      $x^2 - 3 = 0$
      $x = \pm \sqrt{3}$

2. Write the following in the form $a + bi$:
   a) $(3 - 4i)^2 = 9 - 12i - 12i + 16i^2$
      $= 9 - 24i + 16(-1)$
      $= -7 - 24i$

   b) $\frac{4i}{1-2i} \left( \frac{1+2i}{1+2i} \right)$
      $= \frac{4i + 8i^2}{1 - 2i + 2i - 4i^2}$
      $= \frac{4i + 8(-1)}{1 - 4(-1)}$
      $= \frac{-8 + 4i}{5}$
      $= \frac{-8}{5} + \frac{4}{5}i$

      or $\frac{-8}{5} + \frac{4}{5}i$
3. Find the equation of the line parallel to $2x - 4y = 3$ which passes through $(6, -3)$.

$-4y = -2x + 3$

$y = \frac{1}{2}x - \frac{3}{4}$

$m = \frac{1}{2}$

$y - (-3) = \frac{1}{2}(x - 6)$

$y + 3 = \frac{1}{2}x - 3$

$y = \frac{1}{2}x - 6$

4. Let $f(x) = x^2 - 4$ and $g(x) = \frac{x}{2x + 6}$. Find and simplify:

a) $(f - g)(-2) = f(-2) - g(-2)
   = ((-2)^2 - 4) - \left(\frac{-2}{2(-2)+6}\right)
   = 0 - (-1) = 1$

b) $(f \circ g)(0) = f(g(0))
   = f\left(\frac{0}{6}\right) = f(0) = 0 - 4 = -4$

c) $(g \circ f)(x) = g(f(x)) = g(x^2 - 4)
   = \frac{x^2 - 4}{2(x^2 - 4) + 6}
   = \frac{x^2 - 4}{2x^2 - 2}$

d) The domain of $g(x)$ in interval notation

$2x + 6 = 0$

$2x = -6$

$so \quad x \neq -3$

$D: (-\infty, -3) \cup (-3, \infty)$

e) The domain of $f(x)$ in interval notation

$f(x)$ is a polynomial, so

$D: (-\infty, \infty)$
5. Simplify the difference quotient \( \frac{f(x+h) - f(x)}{h} \) when \( f(x) = 4x + 2 \).

\[
\begin{align*}
(1) \quad f(x+h) &= 4(x+h) + 2 = 4x + 4h + 2 \\
(2) \quad 4x + 4h + 2 - (4x+2) &= 4h \\
(3) \quad \frac{4h}{h} &= 4
\end{align*}
\]

6. Find the relative maximum and/or minimum of the function \( G(x) = 4x - 4x^3 \) and the interval(s) on which \( G(x) \) is increasing.

\[
\begin{align*}
\text{Min.} & \quad (-1.57735, -1.5396) \\
\text{Max.} & \quad (-1.57735, 1.5396)
\end{align*}
\]

Is this function even, odd, or neither? Justify your answer using algebra!

\[
G(-x) = 4(-x) - 4(-x)^3 = -4x + 4x^3 = - (4x - 4x^3) = -G(x)
\]

7. Solve the quadratic equation \( x^2 - 6x - 3 = 0 \) by completing the square.

\[
\begin{align*}
x^2 - 6x &= 3 \\
x^2 - 6x + 9 &= 12 \\
(x-3)^2 &= 12 \\
x-3 &= \pm \sqrt{12} \\
x &= 3 \pm 2\sqrt{3}
\end{align*}
\]

8. Find the equation of a function which resembles \( y = x^3 \) shifted right 3 units and down 7 units.

\[
y = f(x) = (x-3)^3 - 7
\]
9. A stone is thrown upward from a height of 100 feet with an initial velocity of 80 ft/sec. The height of the stone t seconds after it has been thrown is given by the function \( s(t) = -16t^2 + 80t + 100 \). Find the maximum height of the stone. 

\[ t = \frac{-80}{2(-16)} = \frac{5}{2} = 2\frac{1}{2} \text{ seconds} \]

\[ s(\frac{5}{2}) = 200 \text{ ft}. \]

10. Are the lines given by the equations \( 2x + y = 4 \) and \( 2x - 4y = 6 \) perpendicular to each other? Why or why not?

\[ y = -2x + 4 \quad -4y = -2x + 6 \]

\[ y = \frac{1}{2}x - \frac{3}{2} \]

Yes; the slopes are negative reciprocals.

11. Sketch a graph of the function \( f(x) = 4x - 6, \ x \leq 3 \quad 3 - x, \ x > 3 \)

Label at least 3 points.

\[
\begin{array}{c|c|c}
 x \leq 3 & y = 4x - 6 & \\
 0 & -6 & \\
 3 & 6 & \\
 x > 3 & y = 3 - x & \\
 \text{hole} & 3 & 0 \\
 4 & -1 & \\
 5 & -2 & \\
\end{array}
\]