1. Let $G$ be a simple $n$ vertex graph with $n/2 - 1 \leq \delta(G) \leq n - 2$. Prove that $G$ is $k$-connected for all $k$ with $k \leq 2\delta(G) + 2 - n$. Prove that this is best possible for all $\delta \geq n/2 - 1$ by constructing a simple $n$ vertex graph with minimum degree $\delta$ that is not $k$-connected for $k = 2\delta+3-n$.

2. Let $G$ be a $k$-connected graph and let $S, T$ be disjoint subsets of $V(G)$ with the size each at least $k$. Prove that $G$ has $k$ pairwise disjoint $S,T$-paths.

3. Let $X$ and $Y$ be disjoint sets of vertices in a $k$-connected graph $G$. Let $u(x)$ for $x \in X$ and $w(y)$ for $y \in Y$ be nonnegative integers such that $\sum_{x \in X} u(x) = \sum_{y \in Y} w(y) = k$. Prove that $G$ has $k$ pairwise internally disjoint $X,Y$-paths so that $u(x)$ of them start at $x$ and $w(y)$ of them end at $y$, for all $x \in X$ and $y \in Y$.

4. Show that Menger’s Theorem implies the König-Egerváry Theorem.