Instructions: Show all of your work and clearly explain your answers. No books or written notes are allowed during the exam. Do any 4 of the 5 problems. Use one page per problem. Be sure to clearly indicate which of the 5 problems you want graded. Each problem counts 25 points, for a total of 100.

[1] Let \( S \) be the set \( \{a, b, c\} \).
(a) Define a relation on the set \( S \) by putting checkmarks in a labeled tic-tac-toe grid, as on the board. Check as many squares as possible, such that the relation you define is NOT reflexive.
(b) This time let the set \( S \) be the set of all people. Say person \( A \) is related to person \( B \) if \( A \) and \( B \) share a grandparent, but not a parent (i.e., if \( A \) and \( B \) are cousins but not siblings). For each of the properties reflexivity, symmetry and transitivity, determine whether or not the property holds for this relation. Justify your answer in each case.

[2] Prove the formula \( 1 + 3 + 5 + \cdots + (2n - 1) = n^2 \) for each \( n \geq 1 \). (I.e., prove that the sum of the first \( n \) odd integers equals \( n^2 \).) Include each step of the proof in your answer.

[3] Let \( a_1 = 3, \ a_2 = 5, \) and define \( a_{n+1} = 3a_n - a_{n-1} \) for \( n \geq 2 \). Prove that \( a_k > 2^k \) for each integer \( k \geq 1 \).

[4]
(a) Use Euclid’s method to find \( \gcd(357, 918) \).
(b) Show how to use your work in (a) to find an integer solution to \( 918x + 357y = \gcd(357, 918) \).
(c) Find the least positive integer \( y \) such that \( 918x + 357y = \gcd(357, 918) \) has a solution where \( x \) also is an integer. Justify your answer.

[5]
(a) Explain why the least positive integer linear combination \( 111x + 74y \) of 111 and 74 is 37.
(b) Let \( k \) be an integer. Justify why \( 111x + 74y = k \) has a solution if and only if \( 37 | k \).