1. Evaluate \( \int \frac{5x^2 - 10x + 17}{(x-1)(x^2 - 2x + 5)} \, dx \) using partial fraction decomposition.

**Solution.** Notice that \( x^2 - 2x + 5 = (x^2 - 2x + 1) + 4 = (x-1)^2 + 4 \) has no real roots, the denominator is already factored. Thus, the form of the partial fractions decomposition is

\[
\frac{5x^2 - 10x + 17}{(x-1)(x^2 - 2x + 5)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 - 2x + 5}.
\]

Multiplying both sides by \( (x-1)(x^2 - 2x + 5) \), we get

\[
5x^2 - 10x + 17 = A(x^2 - 2x + 5) + (Bx + C)(x-1).
\]

Setting \( x = 1 \), we get

\[
5 \cdot 1^2 - 10 \cdot 1 + 17 = A(1^2 - 2 \cdot 1 + 5)
\]

\[
12 = 4A
\]

and so \( A = 3 \).

Setting \( x = 0 \), we get \( 17 = 5A + C(-1) \) and, since \( A = 3 \), \( C = 5A - 17 = -2 \).

Finally, we let \( x = -1 \) to get

\[
5 + 10 + 17 = 10A + (-B + C)(-2)
\]

\[
32 = 10(3) + (-B - 2)(-2)
\]

\[
2 - 4 = -B
\]

and so \( B = 2 \).

Thus, we have

\[
\int \frac{5x^2 - 10x + 17}{(x-1)(x^2 - 2x + 5)} \, dx = \int \frac{3}{x-1} + \frac{2x - 2}{x^2 - 2x + 5} \, dx
\]

\[
= 3 \ln |x - 1| + \int \frac{1}{u} \, du \quad u = 2x^2 - 2x + 5, \, du = 2x - 2 \, dx
\]

\[
= 3 \ln |x - 1| + \ln |u| + C
\]

\[
= 3 \ln |x - 1| + \ln |x^2 - 2x + 5| + C
\]