1. Evaluate \( \int \frac{7x^3 - 12x^2 - 6x + 19}{(x-2)(x+3)(x^2 - 4x + 5)} \, dx. \)

To evaluate one of the integrals in the partial fractions decomposition, you might want to complete the square and substitute for \( x - 2. \)

**Solution.** Notice that \( x^2 - 4x + 5 = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1; \) this is an irreducible quadratic factor since it has no real roots.

The form of the partial fractions decomposition is

\[
\frac{7x^3 - 12x^2 - 6x + 19}{(x-2)(x+3)(x^2 - 4x + 5)} = \frac{A}{x-2} + \frac{B}{x+3} + \frac{Cx + D}{(x-2)^2 + 1}.
\]

Multiplying by \( (x-2)(x+3)(x^2 - 4x + 5), \) we get

\[
7x^3 - 12x^2 - 6x + 19 = A(x+3)((x-2)^2 + 1) + B(x-2)((x-2)^2 + 1)
\]
\[
+ (Cx + D)(x-2)(x+3).
\]

Letting \( x = 2, \) we get

\[
7 \cdot 8 - 12 \cdot 4 - 6 \cdot 2 + 19 = A \cdot 5 \cdot 1
\]
\[
15 = 5A
\]

and so \( A = 3. \)

Letting \( x = -3, \) we get

\[
7 \cdot (-27) - 12 \cdot 9 - 6 \cdot (-3) + 19 = B \cdot (-5) \cdot 26
\]
\[
-260 = -130B
\]

and so \( B = 2. \)

Next, we let \( x = 0, \) to get \( 19 = 15A - 10B - 6D. \) Since \( A = 3 \) and \( B = 2, \) we have \( 6D = 15A - 10B - 19 = 45 - 20 - 19 = 6 \) and so \( D = 1. \) Finally, let \( x = 1, \) to get

\[
7 - 12 - 6 + 19 = A \cdot 8 + B \cdot (-2) + (C + D) \cdot (-4)
\]
\[
8 = 8A - 2B - 4C - 4D
\]

Dividing by 4 and solving for \( C, \) \( C = 2A - B/2 - D - 2 = 6 - 1 - 1 - 2 = 2. \) Thus,

\[
\frac{7x^3 - 12x^2 - 6x + 19}{(x-2)(x+3)(x^2 - 4x + 5)} = \frac{3}{x-2} + \frac{2}{x+3} + \frac{2x + 1}{(x-2)^2 + 1}.
\]
Notice \[
\frac{2x + 1}{(x - 2)^2 + 1} = \frac{2(x - 2) + 5}{(x - 2)^2 + 1},
\]
and so using the substitution \( u = x - 2 \), we have
\[
\int \frac{2x + 1}{(x - 2)^2 + 1} \, dx = \int \frac{2u + 5}{u^2 + 1} \, du = \int \frac{2u}{u^2 + 1} \, du + \int \frac{5}{u^2 + 1} \, du = \ln |u^2 + 1| + 5 \arctan(u) + C = \ln |(x - 2)^2 + 1| + 5 \arctan(x - 2) + C.
\]

Thus,
\[
\int \frac{7x^3 - 12x^2 - 6x + 19}{(x - 2)(x + 3)(x^2 - 4x + 5)} \, dx = 3 \ln |x - 2| + 2 \ln |x + 3| + \ln |(x - 2)^2 + 1| + 5 \arctan(x - 2) + C
\]