Last Time:

- polar representation of complex numbers: (r, α).
  - r is taken to be real and positive (or zero).
  - α can be any real number.
- "angles add, magnitudes multiply": $(r, \alpha) \cdot (s, \beta) = (rs, \alpha + \beta)$
- So the n^{th} "roots of unity" are $(1, \frac{2\pi}{n}), (1, \frac{4\pi}{n}), \ldots, (1, \frac{2(n-1)\pi}{n})$. There are exactly n complex n^{th} roots of 1.

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Question:

Is $(1, \frac{10\pi}{3})$ a cube root of 1?

Your work on $4 + 4\sqrt{3}i$ and $\frac{1}{16} + \frac{1}{16}\sqrt{3}i$ hinted at:

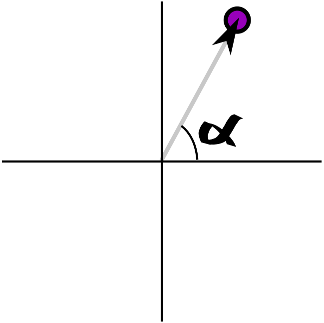
A generalization of the pattern of complex n^{th} roots of 1, to **roots of complex numbers**.

This is an amazing fact: by expanding our view of numbers to complex numbers, we can **always** find **2 square roots**, **3 cube roots**, **4 fourth roots**, **5 fifth roots**, **...** !

The power of **DeMoivre's Theorem** (aka the Holy Grail of Complex Numbers) is making this discovery precise.

DeMoivre's Theorem

The complex n^{th} roots of (R, α) are ...



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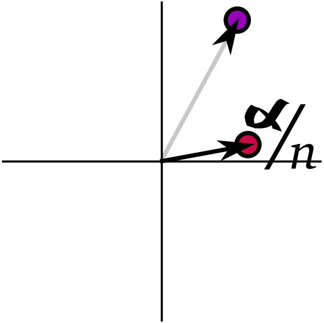

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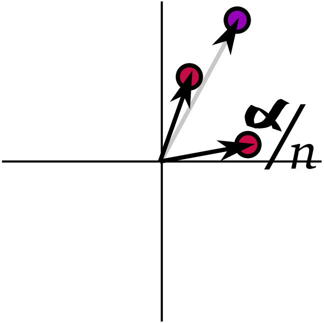

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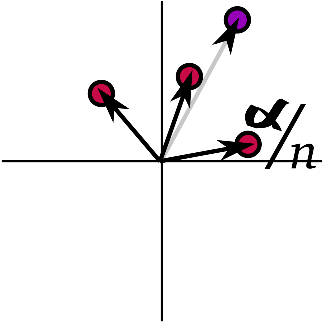

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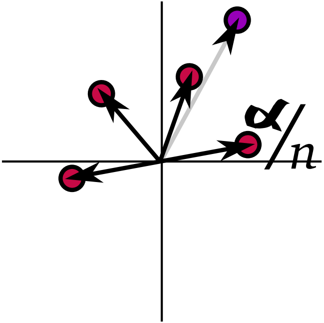

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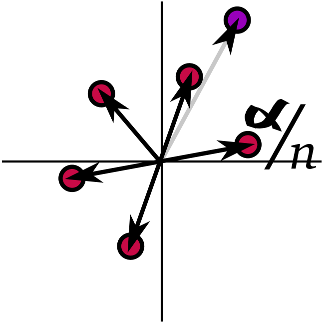

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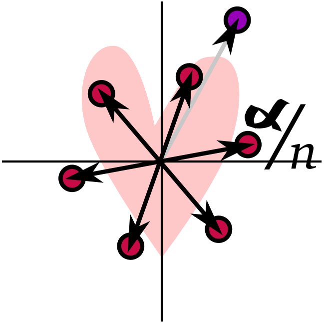

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...,
and $(\sqrt[n]{R}, \frac{\alpha}{n} + \frac{2(n-1)\pi}{n})$.

DeMoivre's Theorem is like a Valentine's Day Card from Geometry to Algebra, each praising the beauty of the other.

Exactly n

Two parts to showing you have **exactly** n:

- You have n different numbers.
- If someone shows you an n^{th} root, it must be one of the ones you found.

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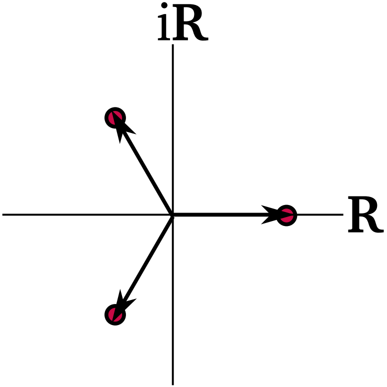
> Please send a representative of your group to pick up the handouts on:
>
> - Features of a Good Explanation
> - DeMoivre's Theorem

Read the first proof of **DeMoivre's Theorem**. In your groups, come up with a good explanation of how each of these parts are accomplished.

Solving even more polynomials

The red points represent solutions to
$z^3 = 1$.



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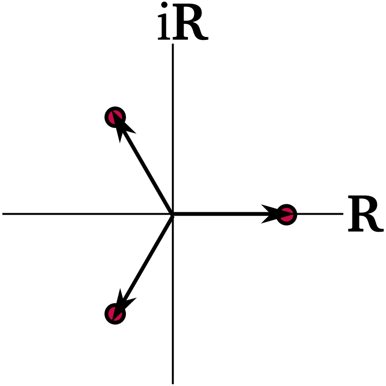

What polynomial's solutions are represented by the blue dots?