# Warm Up

**Please arrange the desks in clusters to satisfy the conditions:**

* You are seated with your March groups
* It is possible to walk easily between clusters
* It is easy to identify who belongs to which cluster.

**Gather:**

* epsilon kits (graphs on pink paper, ε-strip transparencies)

**Discuss:**

* Take a graph of a convergent sequence. Place ε-strips on the graph, aligning the center with the limit of the sequence. How many points lie outside the ε-strip? How many points lie inside? Find out the answer to these questions for each example of a convergent sequence available to you.

**March groups:**

| | | | | | |
|---|---|---|---|---|---|
| **Devin H.**<br>**Keturah C.**<br>**Kayla K.**<br>**Scott H.**<br>**Jim D.** | **Danielle K.**<br>**Jeff V.**<br>**John A.**<br>**Kaitlyn H.**<br>**Carl C.** | **Pete M.**<br>**Rebekah L.**<br>**Jared S.**<br>**Sam B.**<br>**Brian T.** | **Jessica K.**<br>**Laura W.**<br>**Ted W.**<br>**Chris D.**<br>**Patrick M.** | **Kari F.**<br>**Josh S.**<br>**Beth B.**<br>**Brady S.**<br>**Dan E.** | **James R.**<br>**Aaron P.**<br>**Cara C.**<br>**Sam V.**<br>**Megan M.** |

# In/outside of ε-strips centered at the limit

Take a convergent sequence. Place ε-strips on its graph, aligning the center with the limit of the sequence. How many points lie outside the ε-strip? How many points lie inside? Find out the answer to these questions for each example of a convergent sequence available to you, and each ε-strip available to you.

(The sequences (both convergent and divergent) in the packet: )

| | | |
|---|---|---|
| $a_n = \frac{1}{n}$ | $a_n = \sqrt{n}$ | $a_n = \frac{n}{5n}$ |
| $a_n = \begin{cases} 1, & \text{if } n \text{ is odd} \\ 1 - \frac{1}{n}, & \text{if } n \text{ is even} \end{cases}$ | $a_n = 1$ | $a_n = (-1)^n + \frac{1}{n}$ |
| $a_n = \begin{cases} \frac{1}{n}, & \text{if } n \leq 10 \\ \frac{1}{10}, & \text{if } n > 10 \end{cases}$ | $a_n = (-1)^n(1 + \frac{1}{n})$ | $a_n = n^3/2^n$ |
| | $a_n = \begin{cases} 1, & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$ | |

# Tabulating our observations

| | | |
|---|---|---|
| $a_n = \frac{1}{n}$ | $a_n = \sqrt{n}$ | $a_n = \frac{n}{5n}$ |
| $a_n = \begin{cases} 1, & \text{if } n \text{ is odd} \\ 1 - \frac{1}{n}, & \text{if } n \text{ is even} \end{cases}$ | $a_n = 1$ | $a_n = (-1)^n + \frac{1}{n}$ |
| $a_n = \begin{cases} \frac{1}{n}, & \text{if } n \leq 10 \\ \frac{1}{10}, & \text{if } n > 10 \end{cases}$ | $a_n = (-1)^n(1 + \frac{1}{n})$ | $a_n = n^3/2^n$ |
| | $a_n = \begin{cases} 1, & \text{if } n \text{ is odd} \\ \frac{1}{n} & \text{if } n \text{ is even} \end{cases}$ | |

| Convergent sequence | ε | #terms inside ε-strip centered at limit | # terms outside ε-strip centered at limit |
|---|---|---|---|
| | | [on blackboard] | |

# In/outside of ε-strips NOT centered at the limit

(1) Take the sequence **a(n) = 1/n** . Overlap ε-strips on **y = 0.2.** As ε gets smaller, how many points of the graph lie within the strip? How many points lie outside?

(2) What if you overlap ε-strips on **y = −0.2**?

(3) Overlap ε-strips on other sequences, both centered at the limit and not centered at the limit. How many points lie inside/ outside the strip? What happens as ε grows or shrinks?

# Tabulating our observations

| Sequence | Center of ε-strip | Width of ε-strip | #terms inside ε-strip | #terms outside ε-strip |
|---|---|---|---|---|

[on blackboard]

# Two definitions using ε-strips

Send one representative of your group to gather enough definitions handouts for each person in your group.

**Apply Definition A and Definition B to various sequences.**

**Do they properly describe the limit of a sequence?**

**Why or why not?**

**Definition A:** L is a limit of a sequence when infinitely many points on the graph of the sequence are covered by any ε-strip as long as the ε-strip is centered at L.

**Definition B:** L is a limit of a sequence when only finitely many points on the graph of the sequence are NOT covered by any ε-strip as long as the ε-strip is centered at L.

# Tabulating our observations

| Sequence | There is an L that satisfies the conditions of Definition A | There is an L that satisfies the conditions of Definition B |
|---|---|---|

[on blackboard]

# Comparison with the ε-N definition

Let’s zoom in on two definitions:

**The ε-N Definition:** L is the limit of a sequence a(n) if for each $\varepsilon > 0$, there is an N such that $|a(n) - L| < \varepsilon$ for all $n \leq N$. (The N may be different for different ε.)

**The ε-strip Definition B:** L is a limit of a sequence when only finitely many points on the graph of the sequence are NOT covered by any ε-strip as long as the ε-strip is centered at L.

**Write down the ε-N Definition on the bottom of the table you were just working on.**

**Determine which of the sequences in the table satisfy the ε-N Definition. How do you know?**

# Summary of ε-strip activity

Send a representative of your group to gather copies of the ε-strip summary.

In your groups, come up with a description of **how to show that a sequence is convergent.** (This is a description along the lines of the proof by contradiction description: What are the key pieces of the argument? What do you need to define for each piece?)

If you have time, come up with descriptions for --

- **What kinds of divergence are there?**
- **How would someone show that a sequence is divergent?**

# Preview: Properties of the Reals

Next:

**To Infinity and Beyond!**

Featuring …

- Density
- Rationalizing the existence of irrationals
- The return of the **$a+b\sqrt{7}$** family!