Conditions, Solutions, and Conclusions: The Prime Time System Task

Read the following task, suggested by Hyman Bass.

Prime Time System. Let p be an odd prime and let x and y be integers > 0. Suppose that:

$$x + y^2 = 2p \tag{1}$$

$$x^2 + y = 4p. (2)$$

This system of equations (and conditions) has a unique (numerical) solution for (x, y, p). Find it.

(Suggestion: Subtract (1) from (2), and use Euclid's Lemma.)

Find a solution to this task. Submit a summary of your solution online. (There is a link from the cTools site, under Homework Links and Files.)

As you write up your solution, think about the following issues. We will discuss these in class, so please bring your thoughts on these questions.

- How is the word "condition" being used in the statement of the task? What are the conditions to be a solution to the task?
- Where did you use the conditions in your solution?
- How might the solution change if various conditions were eliminated?