Name:_____

Fundamental Theorem of Algebra • Discussion • Math 486-W10

Please form groups of 4 to 6 people, and move your desks so that your group forms a circle.

Discuss the following with your group, and hand this sheet in at the end of class.

Proposition 1 and Proposition 2

- The expression $R^n/2$ shows up in Figure 1, which is a visual schematic for Proposition 1. What is the relationship between $R^n/2$ as it is drawn in Figure 1 and the statement in Proposition 1?

- How does the caption for Figure 2 translate into the statement of Proposition 2?

- Consider the following conversation:

  Student 3: Let's try to summarize Propositions 1 and 2.
  Student 1: OK, Proposition 1 says that image loops are contained in this huge annulus.
  Student 2: And Proposition 2 says that image loops are contained in this small disk.
  Student 3: Wait, I'm confused. How could image loops possibly be contained in both the huge annulus *and* the small disk? Aren't they different regions?

  What information would help here?

- How does the output region in Figure 2 depend on R and f? How does the output region in Figure 1 depend on R and f?

For the questions below, you are encouraged to use your notes from last week.

The Key Idea of Proposition 3

Come up with a visual schematic for the statement of Proposition 3. Once your group reaches a consensus, draw your schematic on the board, along with a caption. Each group member should be prepared to explain how the caption for your figure translates into the statement of Proposition 3.

The Key Idea of Proposition 4

Come up with a visual schematic for the statement of Proposition 4. Once your group reaches a consensus, draw your schematic on the board, along with a caption. Each group member should be prepared to explain how the caption for your figure translates into the statement of Proposition 4.

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Proof of the Fundamental Theorem of Algebra

What are the pieces of the "logical structure" of the proof? In other words, in the proof by contradiction,

- What is the statement that is set up to be contradicted?

- What is the contradiction that was found?

The following sentences from the proof mention Proposition 4.

> *By Proposition 4, there is an annulus $N_\delta(C_R)$ such that if C is a circle in $N_\delta(C_R)$, then $w(f(C)) = w(f(C_R))$. For each R, pick such an annulus and denote it $N(C_R)$.*

- Where does the annulus $N(C_R)$ appear in your schematic of Proposition 4? Does it appear in the input region or output region?

- How many annuli of the form $N(C_R)$ are there?

- Suppose two of these annuli overlap. Why must circles from these annuli produce images with the same winding number?

- How does the proof use overlapping annuli and the assumption that f has no roots to show that all circles with nonzero radius produce images with the same winding number?

Algebra in Proof of Proposition 1 and Proposition 2 (you can do a similar analysis of Proposition 3)

- What is the inequality that Proposition 1 seeks to show?

- What is $\frac{R^n}{2n}$ a bound for? Is it an upper bound or lower bound? How is this bound used in proof of Proposition 1?

- What is the inequality that Proposition 2 seeks to show?

- What is $\frac{|a_0|}{2n}$ a bound for? Is it an upper bound or lower bound? How is this bound used in proof of Proposition 2?

Key Idea of Proof of Proposition 4

- Using conversational language, describe the definition of ϵ_0 in Proposition 4.

- What path is being "shaken around"? What path is it "shaken" to? Why does the "shaking" never cross the origin? What does that tell you about the paths and their winding numbers?