A probabilistic version of the game of zombies and survivors on graphs

Xavier Pérez-Giménez[†]

joint work with

Anthony Bonato[†], Dieter Mitsche[*] and Paweł Prałat[†]

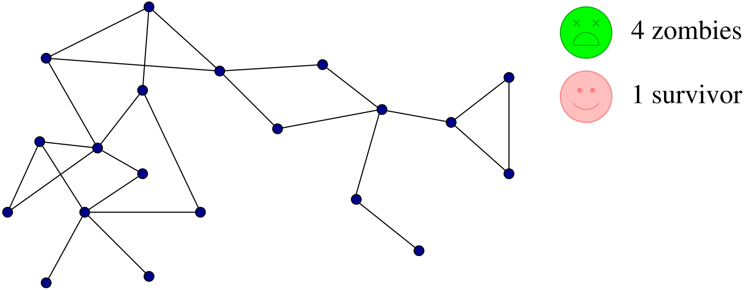
[†]Ryerson University

[*]Université de Nice Sophia-Antipolis

Graphs @ Ryerson, September 2015


Zombie Policeman

Zombies and survivor: who wants to live forever?



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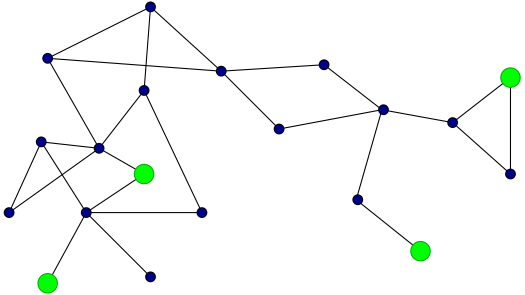

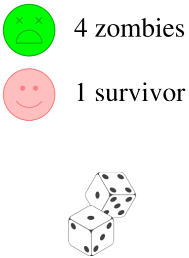

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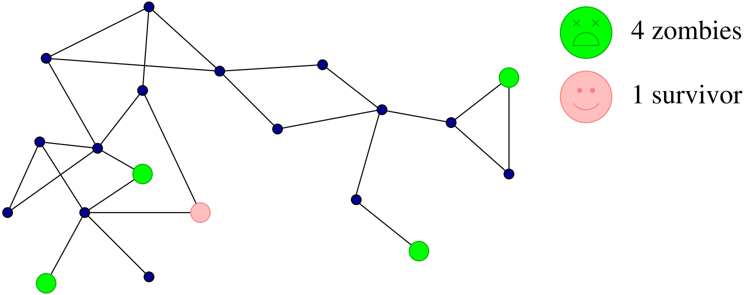

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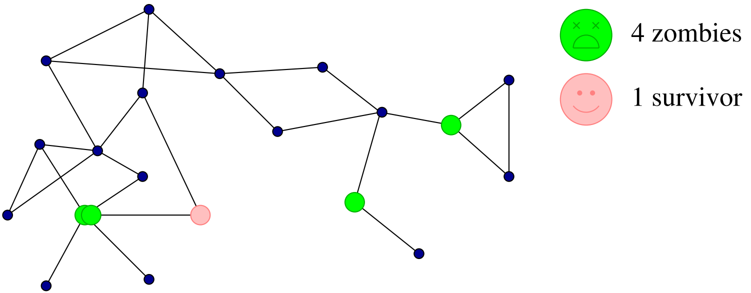

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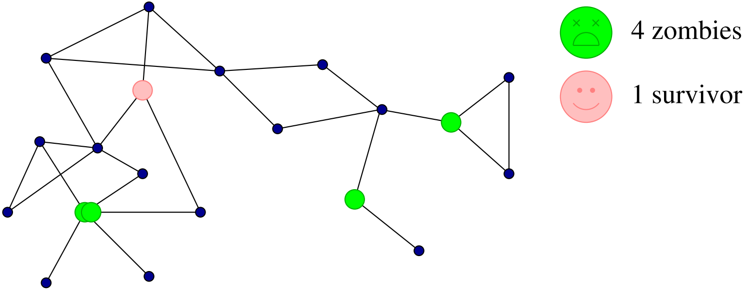

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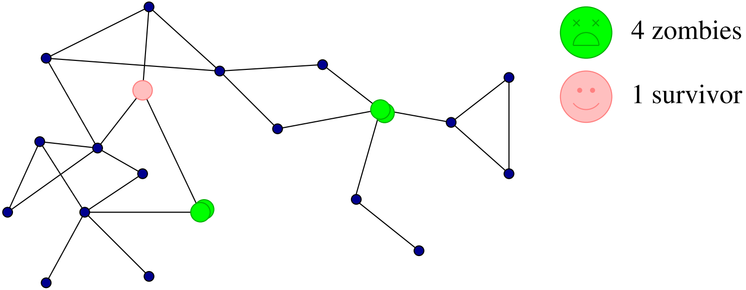

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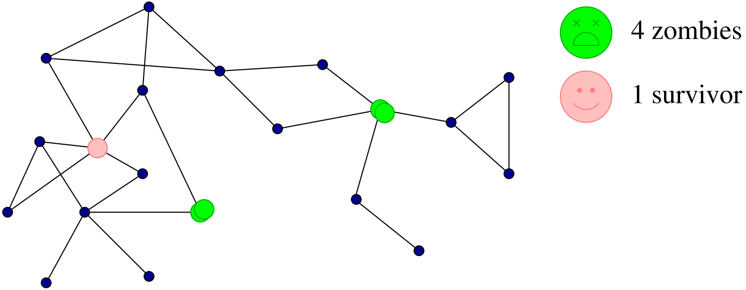

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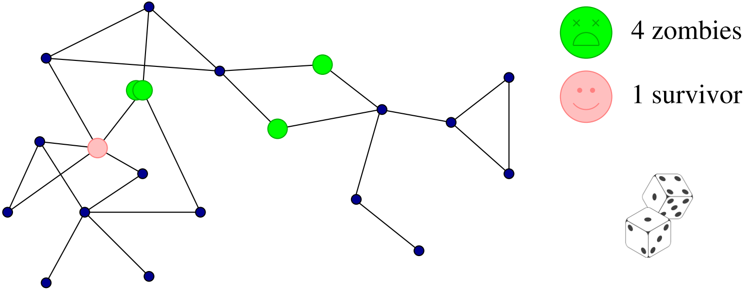

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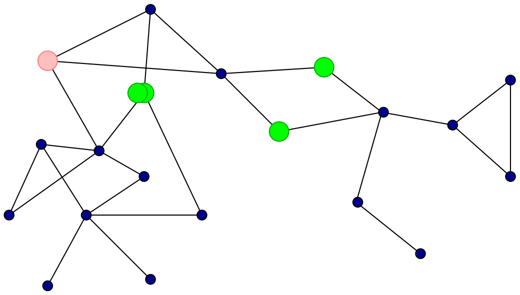

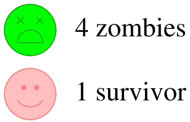

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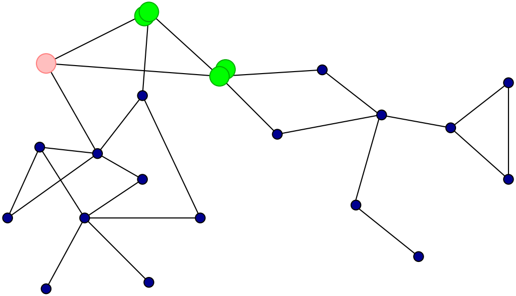

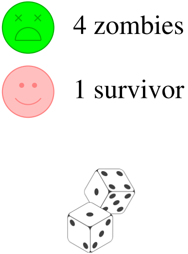

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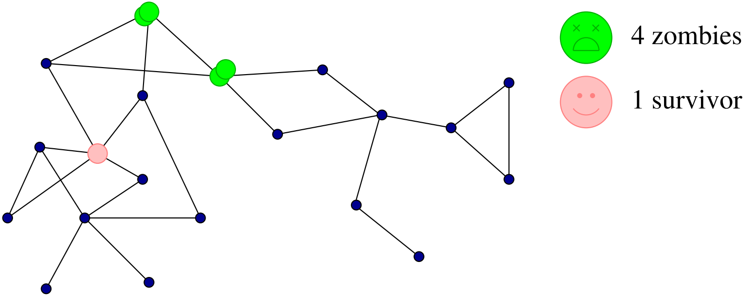

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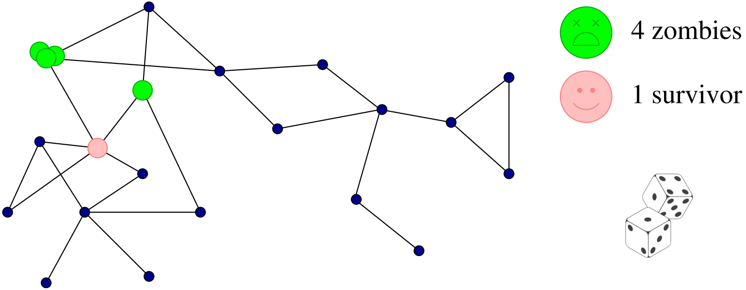

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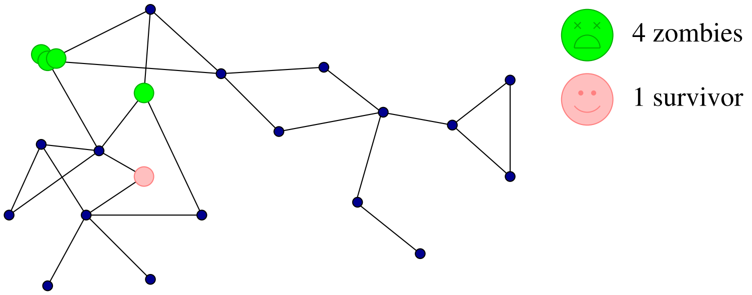

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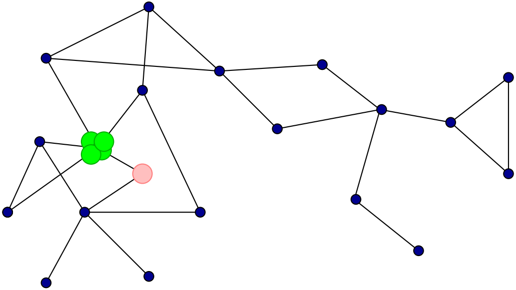

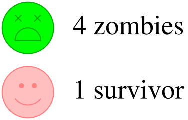

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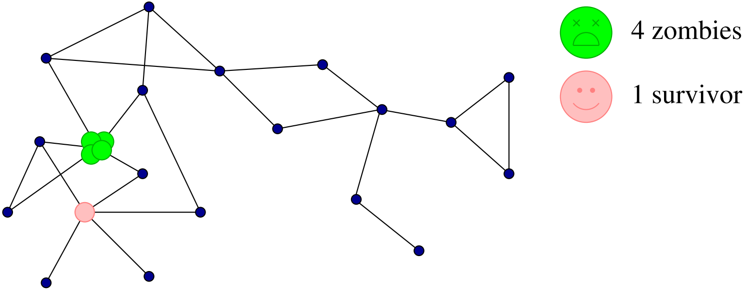

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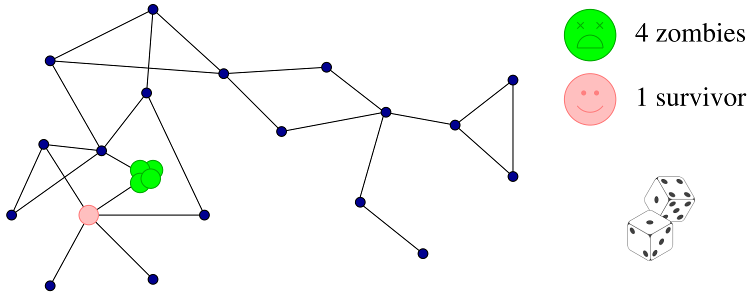

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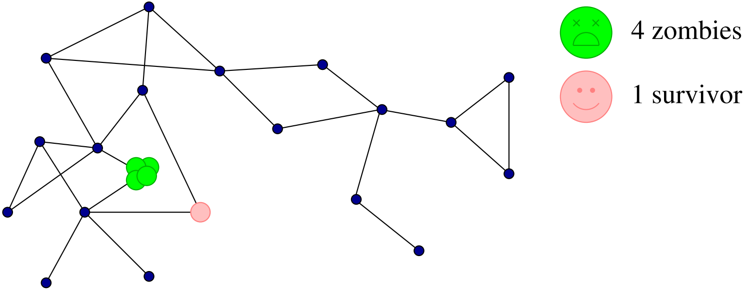

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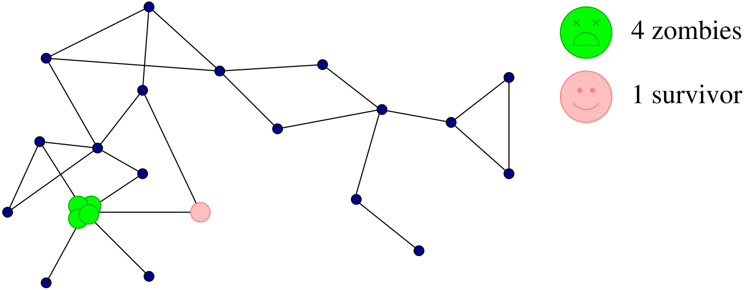

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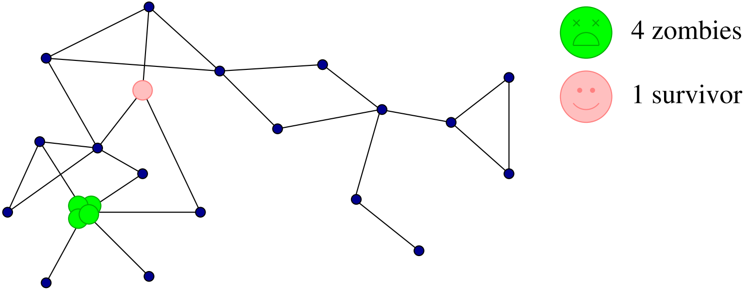

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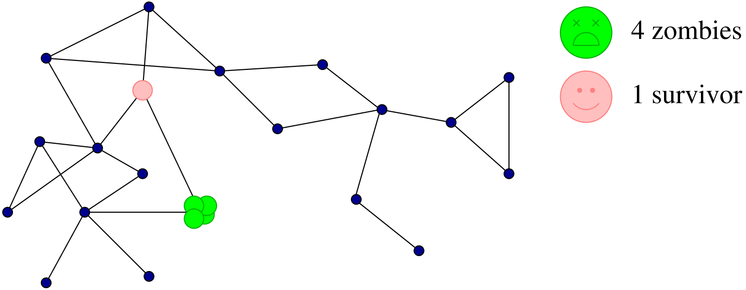

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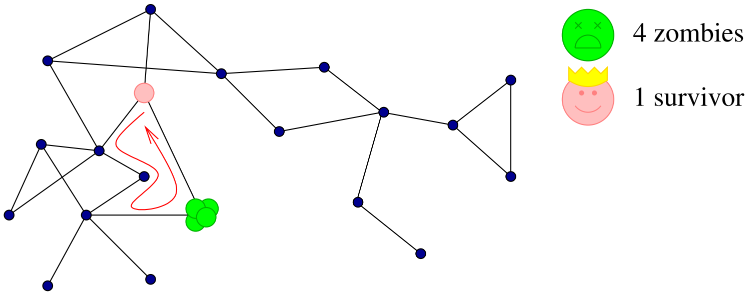

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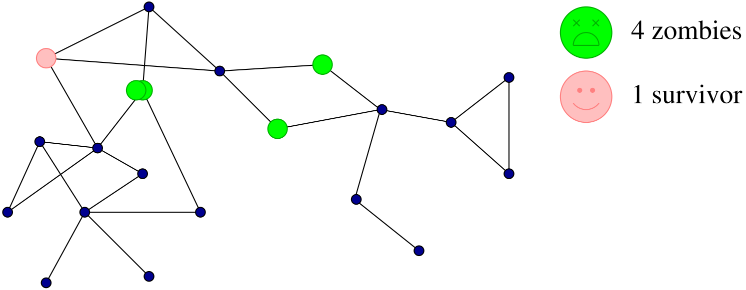

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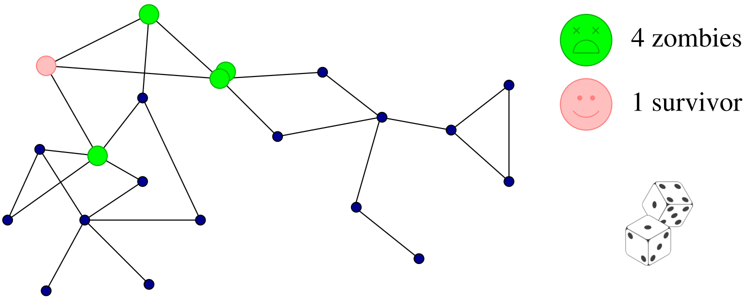

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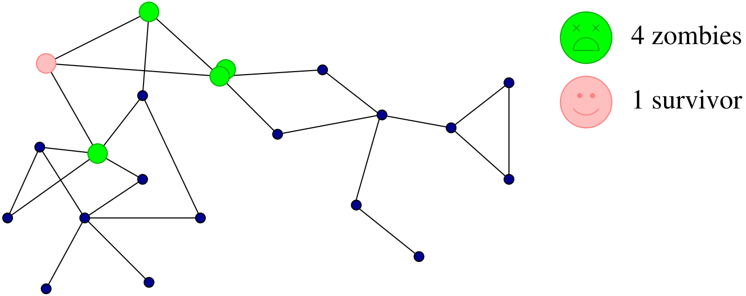

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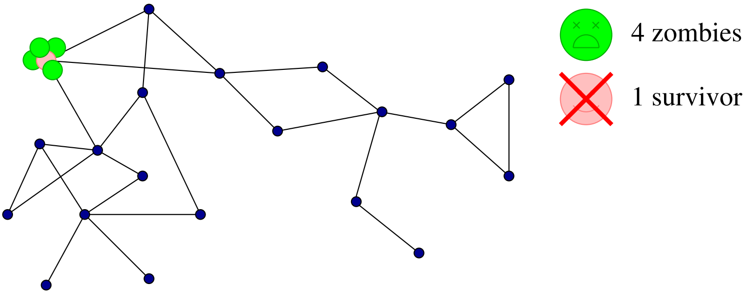

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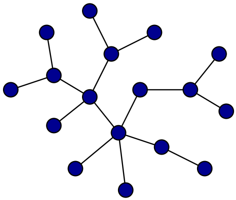

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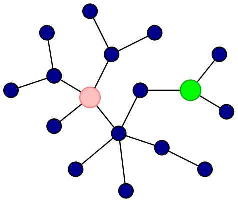

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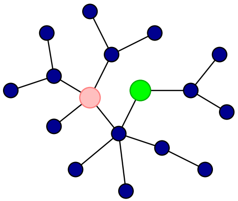

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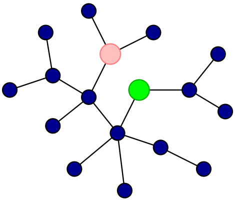

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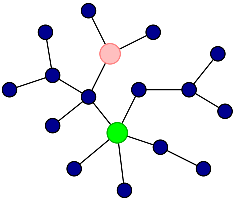

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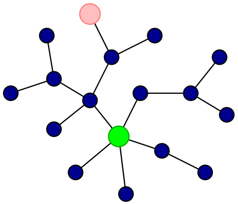

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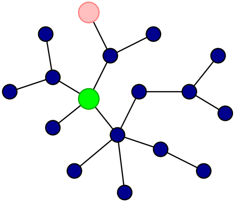

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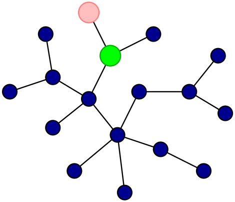

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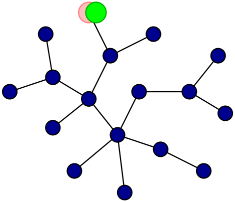

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Price of being undead

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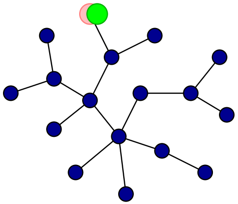

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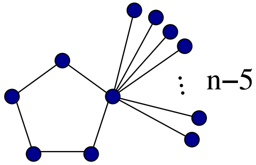

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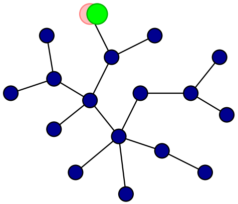

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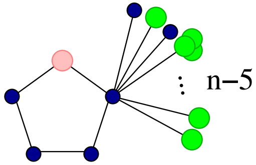

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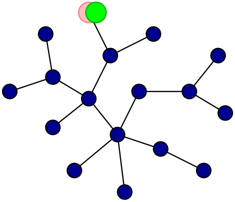

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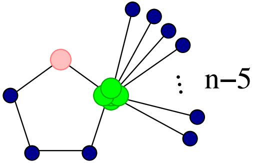

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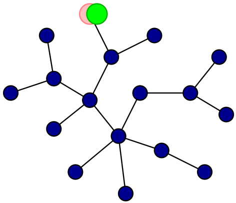

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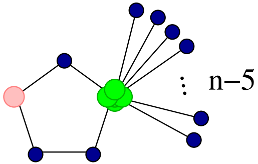

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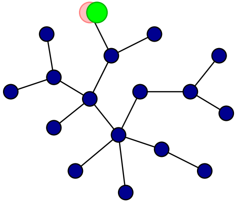

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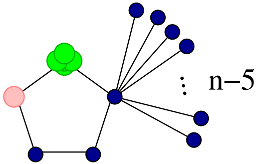

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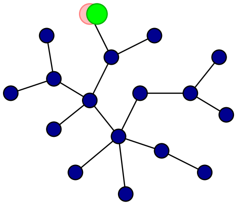

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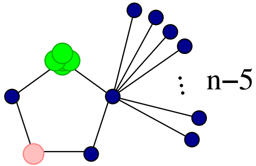

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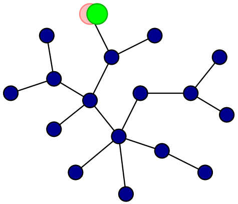

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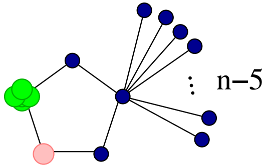

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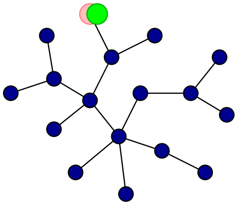

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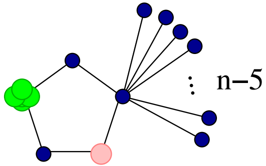

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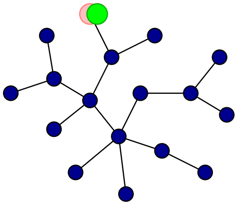

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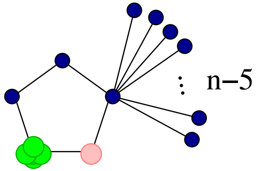

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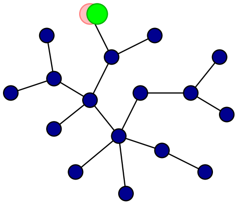

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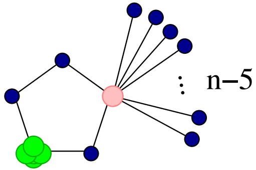

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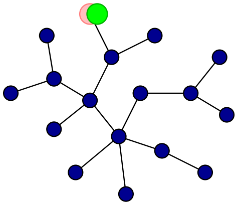

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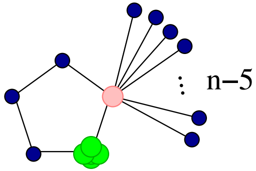

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If $n \geq 27$, then $z(C_n) = 4$ and $Z(C_n) = 2$.

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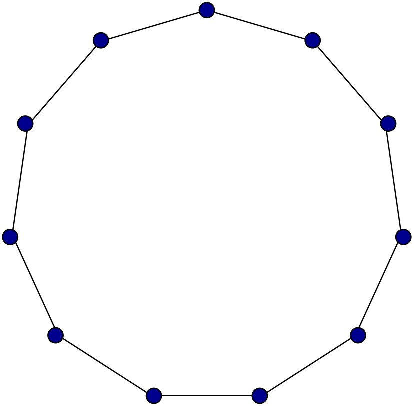

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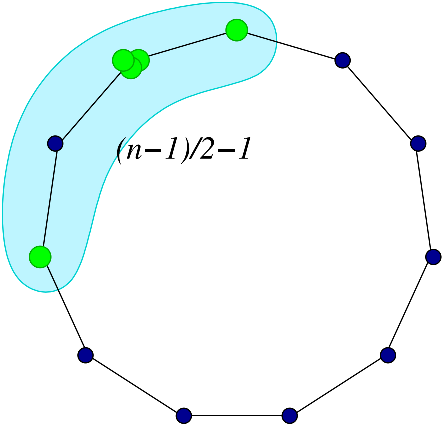

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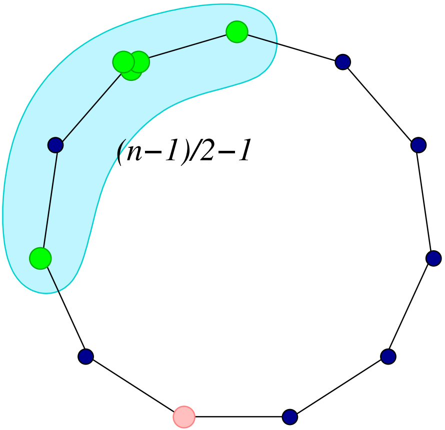

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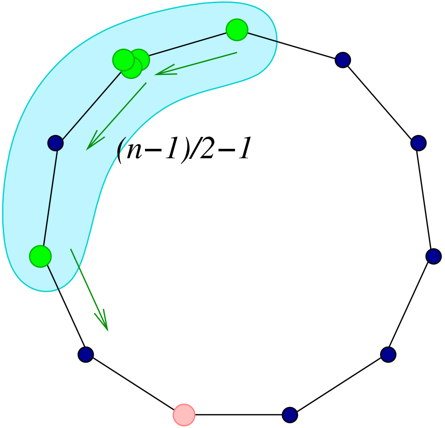

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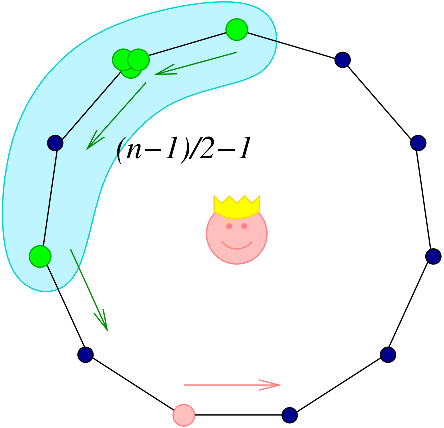

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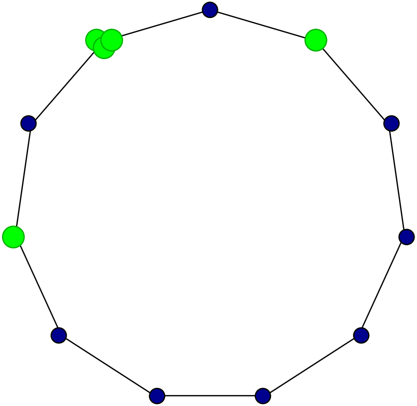

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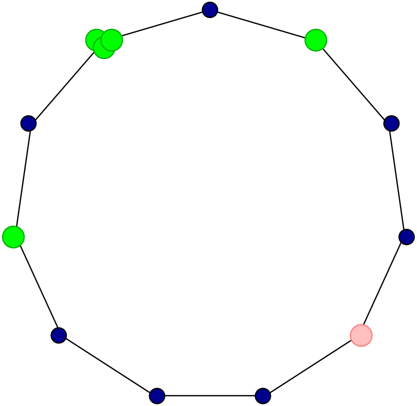

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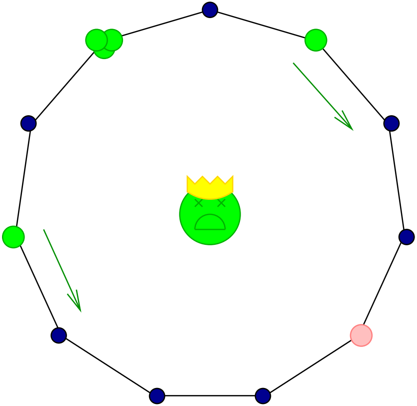

Projective plane P_q of order q
(q prime power)

Graph G_q

Incidence graph of P_q:

Projective plane

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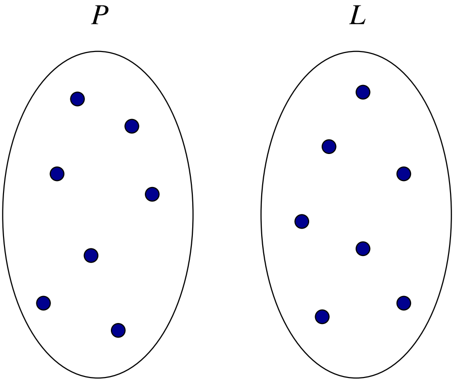
Incidence graph of P_q:

- (P, L)-bipartite



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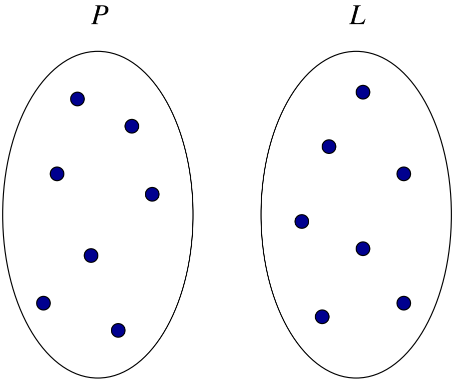

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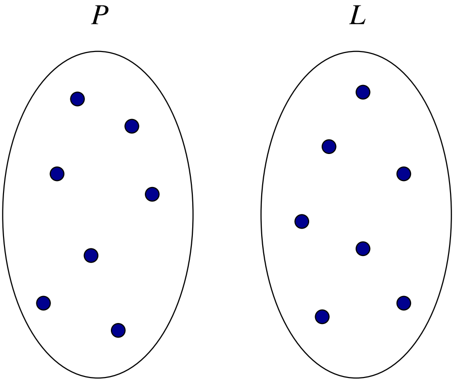

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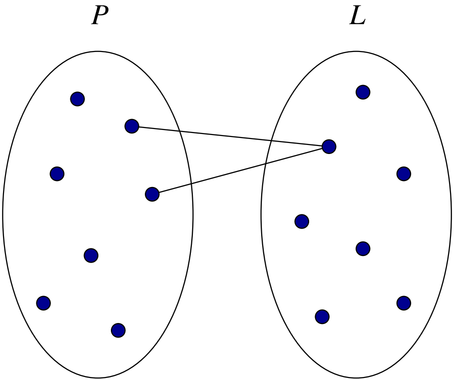

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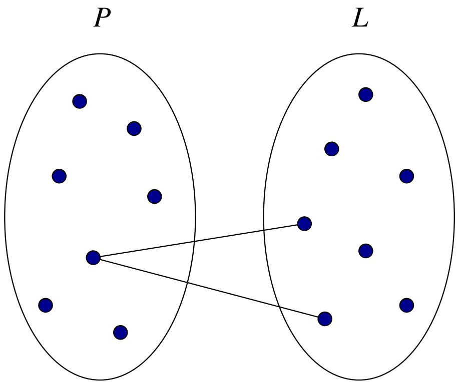

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Lemma

- $k \leq 2q - \omega\sqrt{q} \quad \Longrightarrow \quad$ a.a.s. $\quad k_P, k_L \leq q - 1$

  (survivor strategy).

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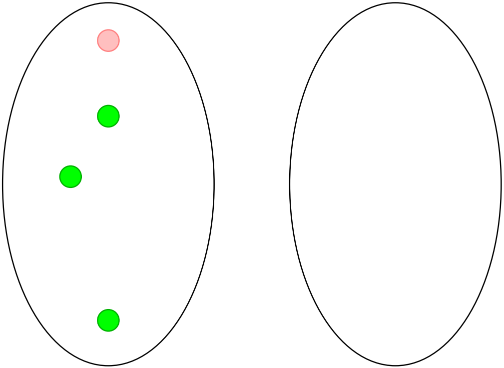
Lemma

- $k \leq 2q - \omega\sqrt{q} \implies$ a.a.s. $k_P, k_L \leq q - 1$ (survivor strategy).
- $k \geq 2q + \omega\sqrt{n} \implies$ a.a.s. $k_P, k_L \geq q$ (zombie strategy).

Projective plane: observation



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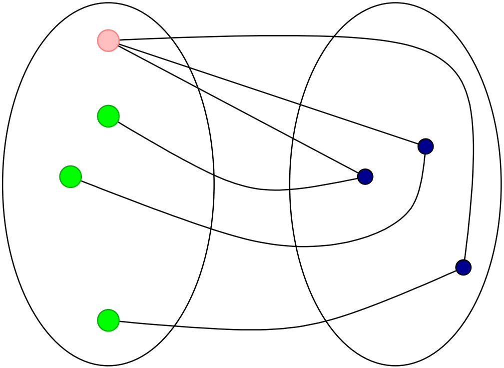

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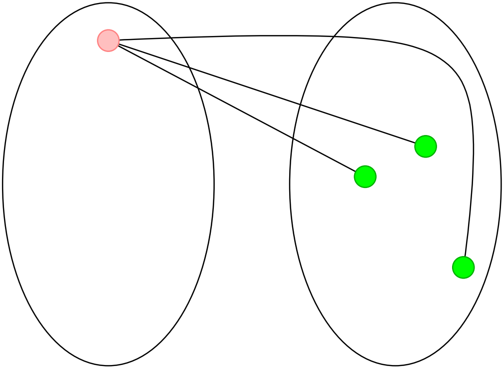

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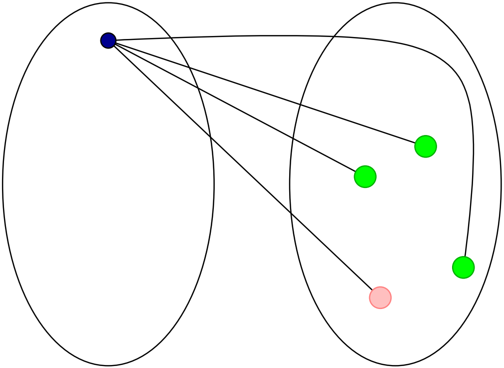

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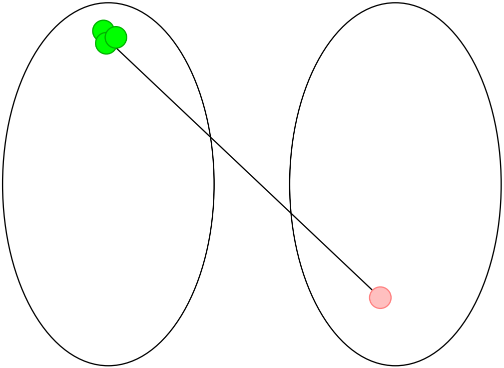

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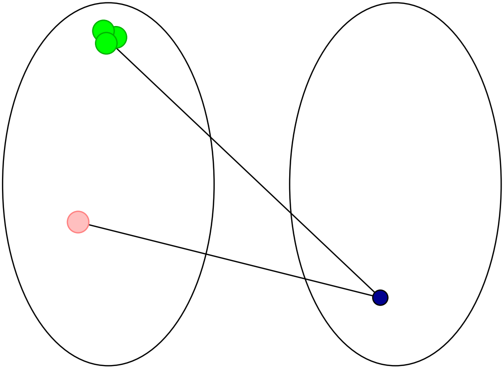

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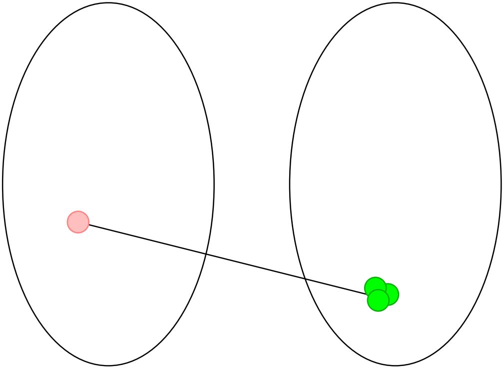
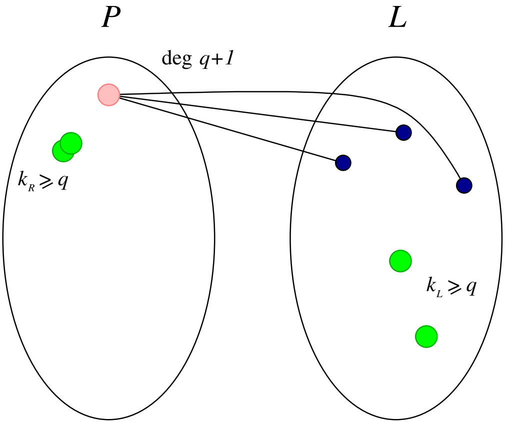


Survivor cannot stop!

Projective plane: zombies' strategy

It's zombies' turn to move...



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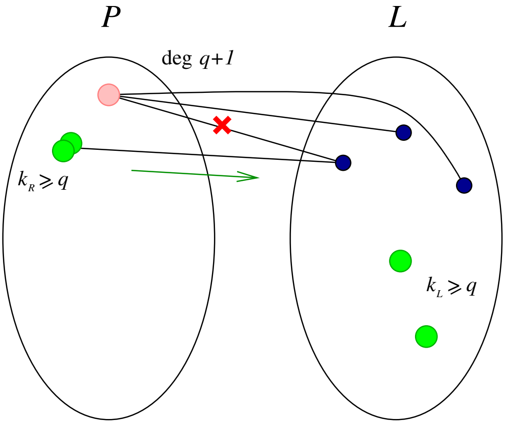

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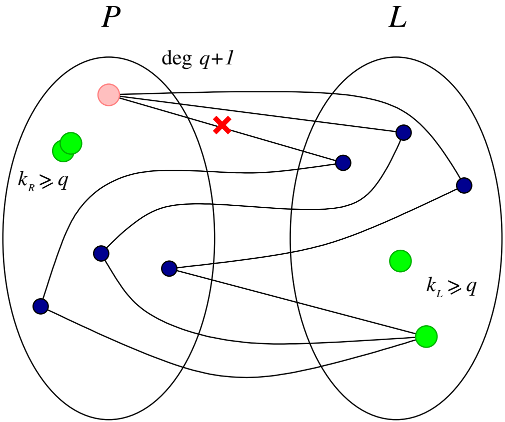

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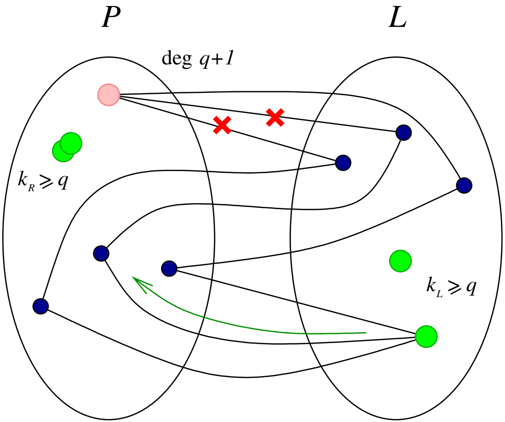

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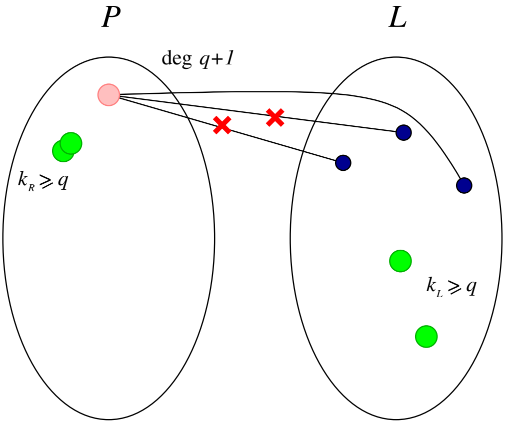

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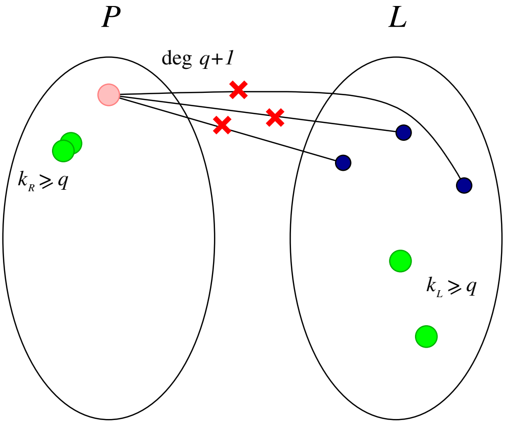

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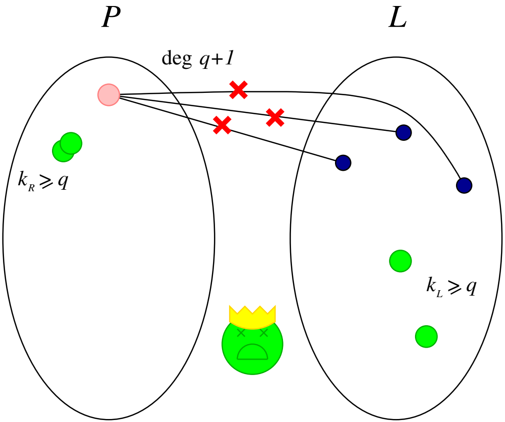

Zombies block all ways of escape with positive probability!

Projective plane: survivor's strategy

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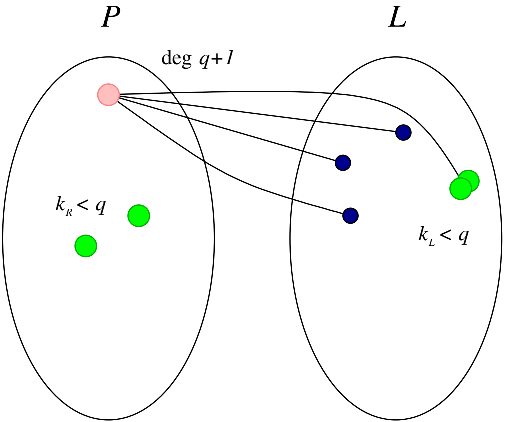

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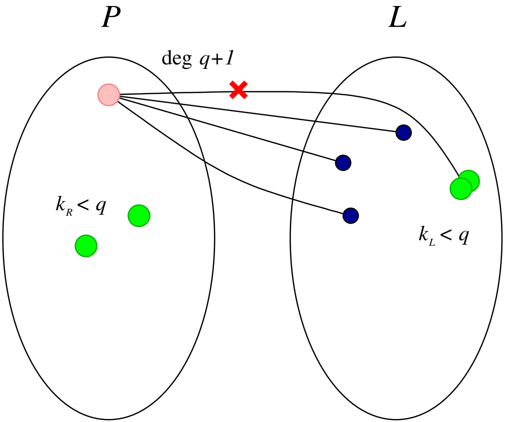

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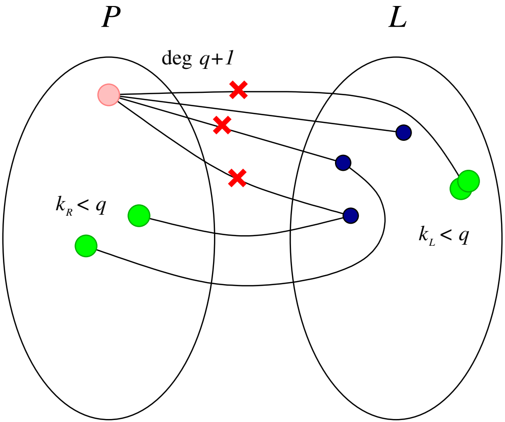

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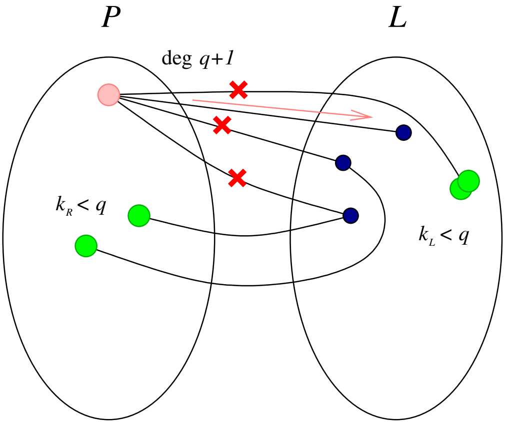

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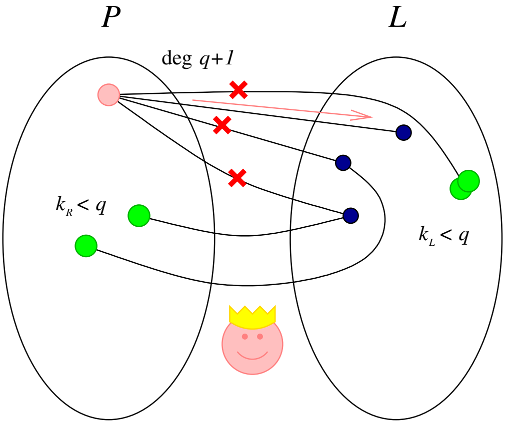
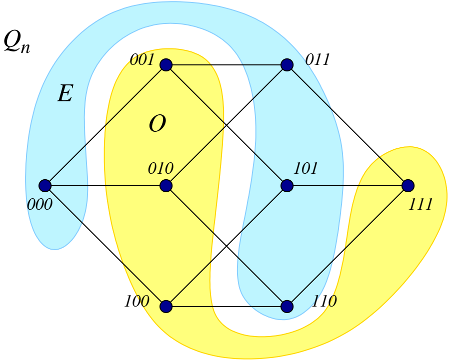


The survivor can always escape for one more step!

Hypercube



- Vertices of Q_n are $\{0,1\}$-strings of length n.
- Q_n is (E,O)-bipartite
- E strings with even number of 1's.
- O strings with odd number of 1's.

Hypercube

Theorem

$z(Q_n) = \frac{2n}{3} + \Theta(\sqrt{n}).$ $\qquad Z(Q_n) \sim 4/3.$

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- $k \geq \frac{2n}{3} + \omega\sqrt{n} \implies$ a.a.s. $k_E, k_O > \frac{n}{3}$ (zombie strategy).

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(The survivor can always find a safe start.)

S_i = set of zombies at distance i from survivor ($|S_1|, |S_2| < n/3$)

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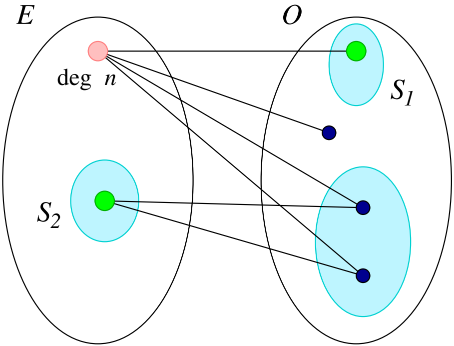
- Otherwise, survivor can find a move away from S_1, S_2.



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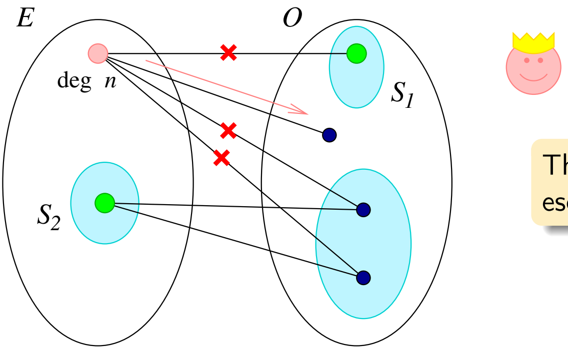

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Vector of distances $\vec{d} = (d_1, \ldots, d_k)$

It never increases (after each zombie move).

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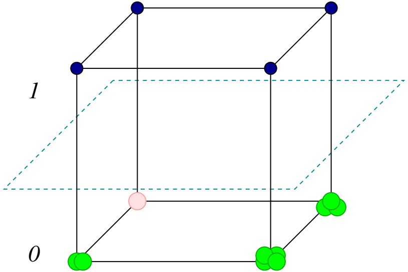

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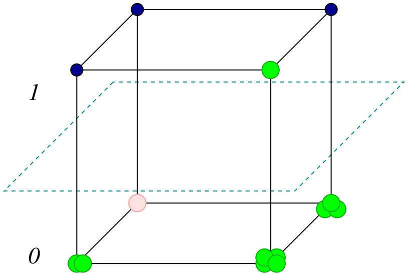

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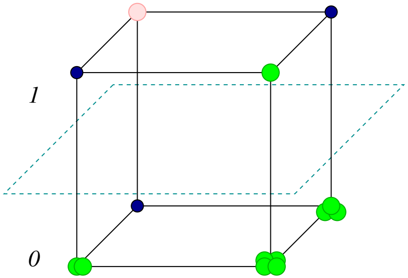

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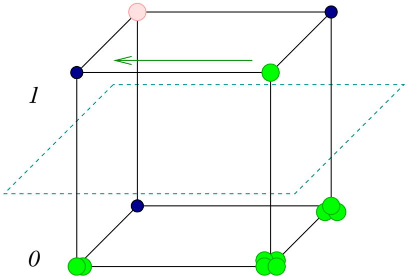

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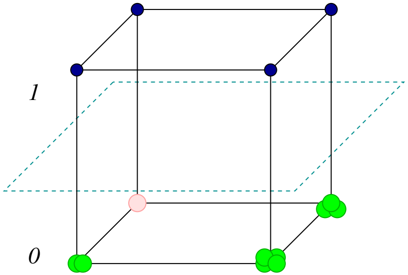

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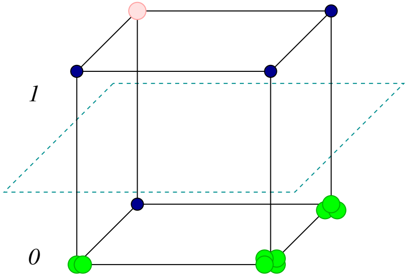

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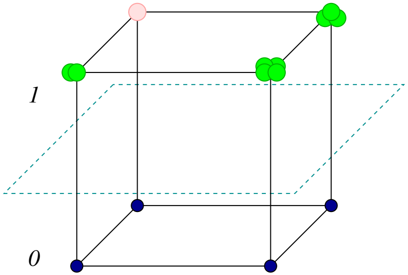

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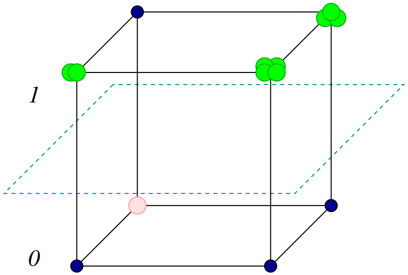

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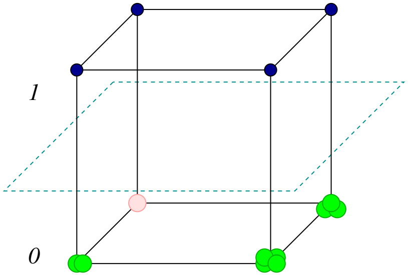

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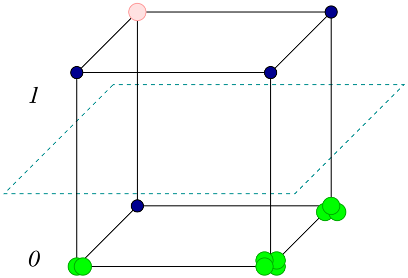

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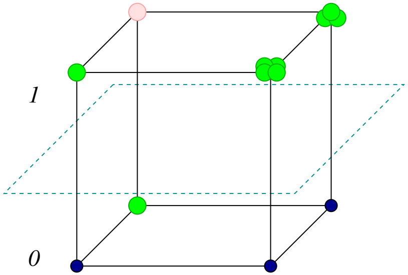

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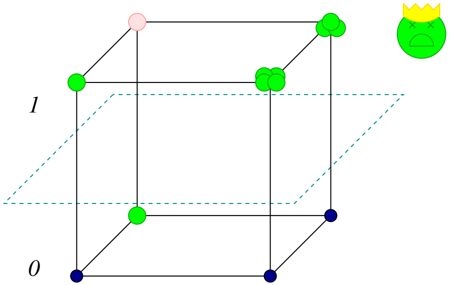
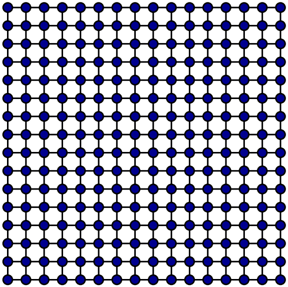


Every n steps, $\vec{d}$ decreases with positive probability.

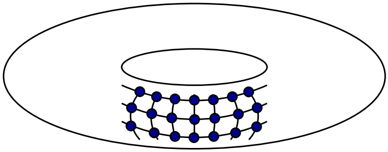
Grids

G_n



$n \times n$ square grid

T_n



$n \times n$ toroidal grid

Grids

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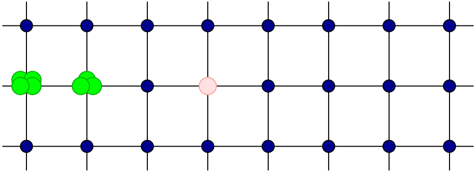

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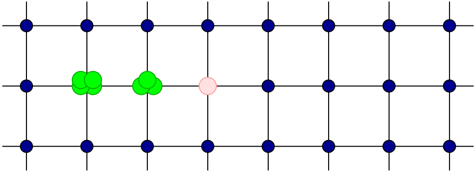

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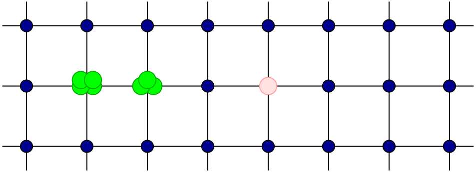

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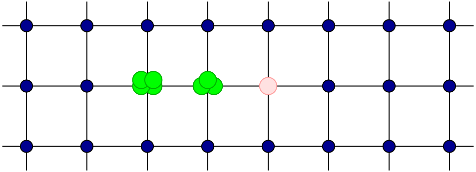

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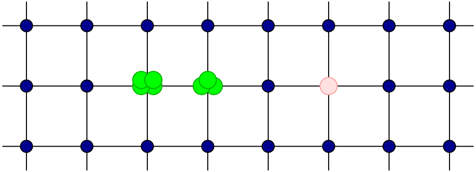

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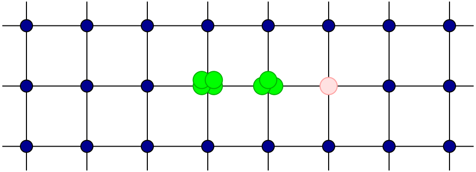

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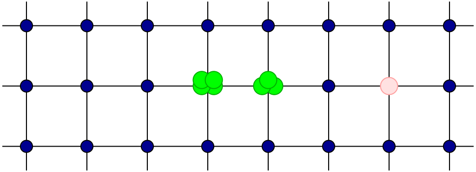

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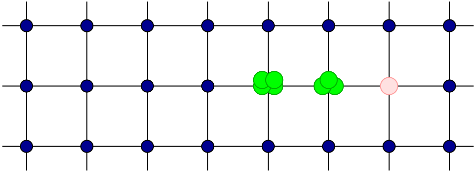

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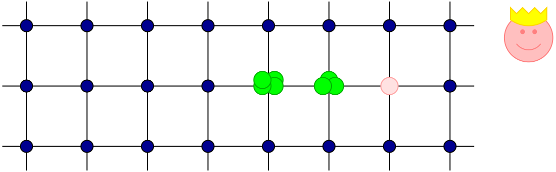
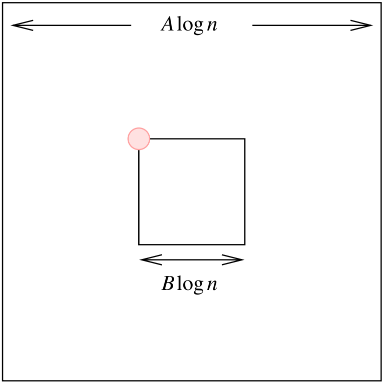


Torus: survivor's strategy



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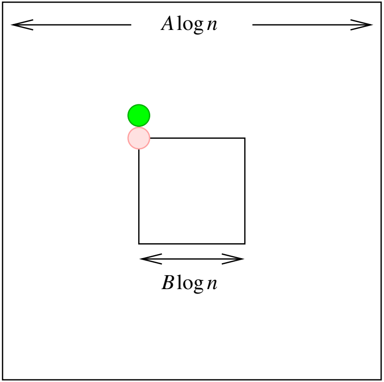

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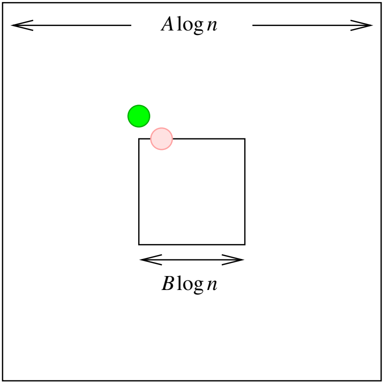

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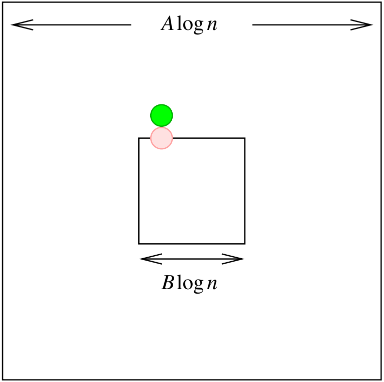

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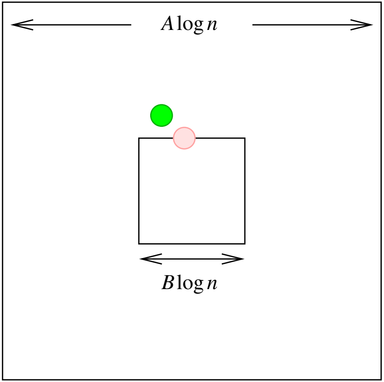

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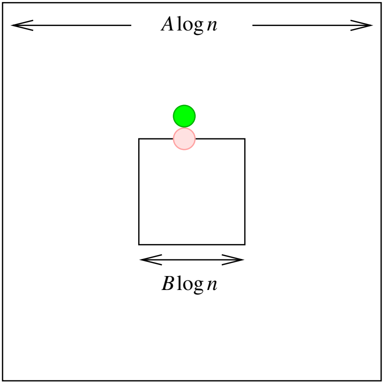

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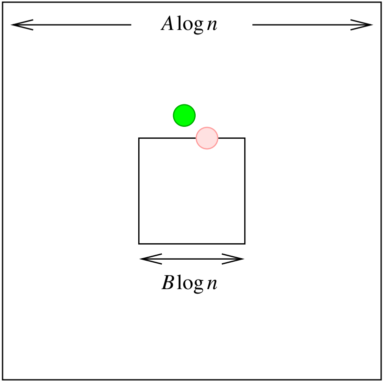

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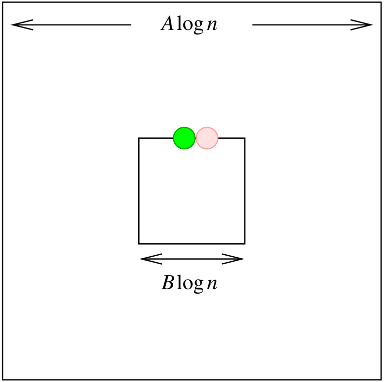

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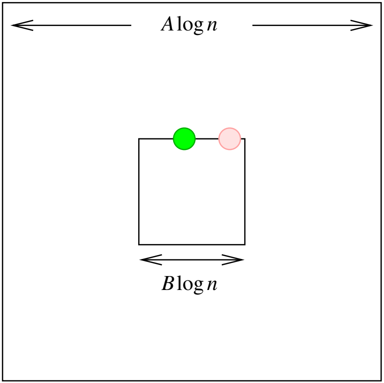

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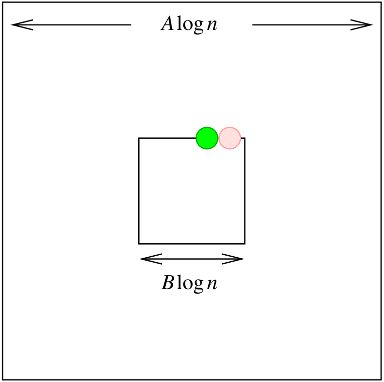

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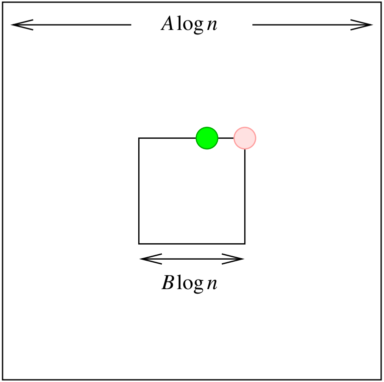

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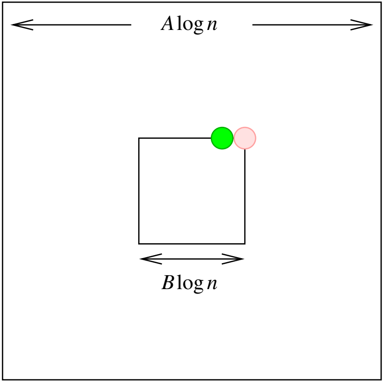

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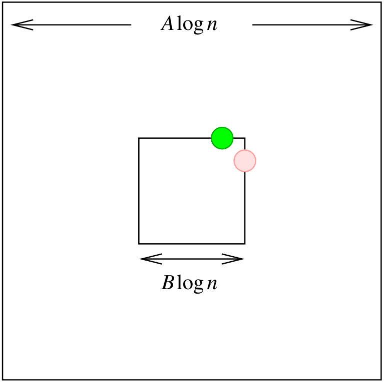

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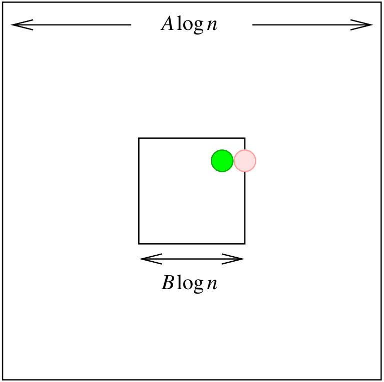

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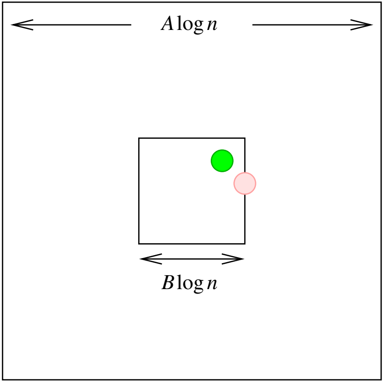
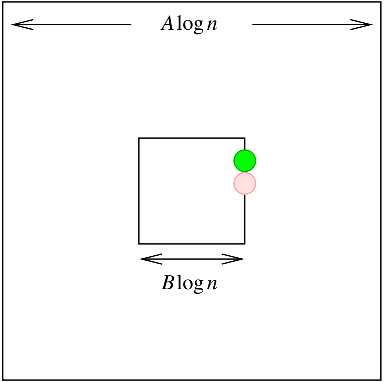
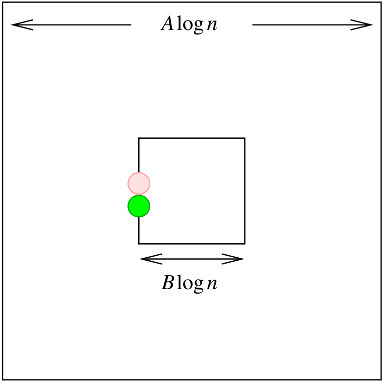


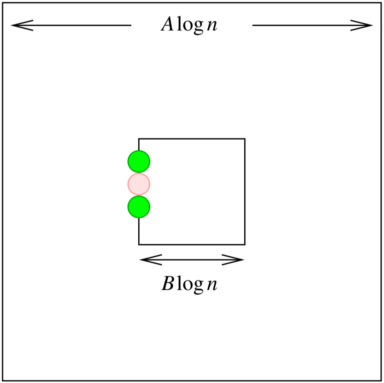
Torus: survivor's strategy



Torus: survivor's strategy



Torus: survivor's strategy



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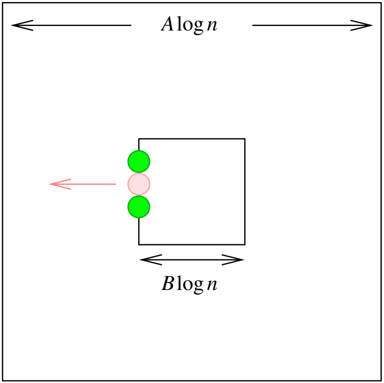
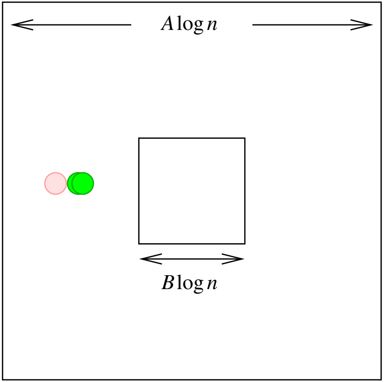
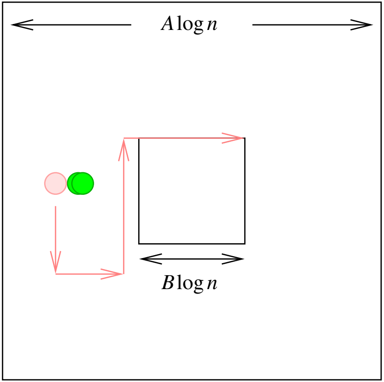


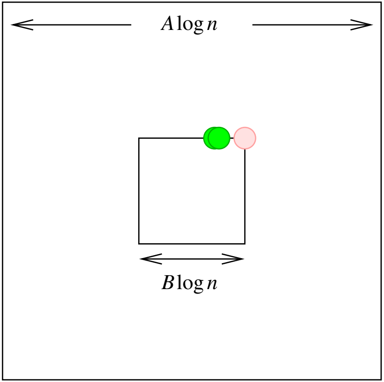
Torus: survivor's strategy



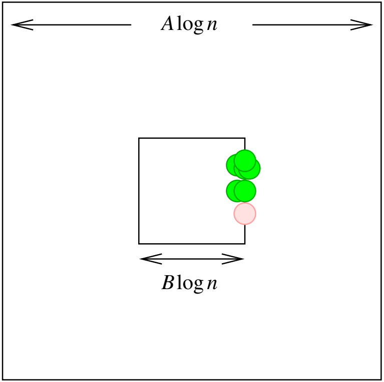
Torus: survivor's strategy



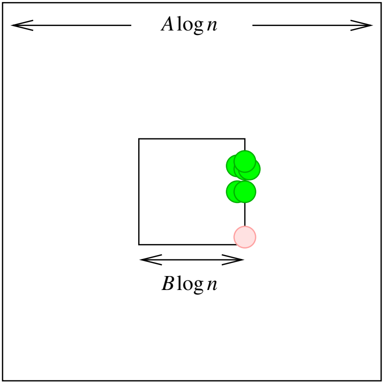
Torus: survivor's strategy



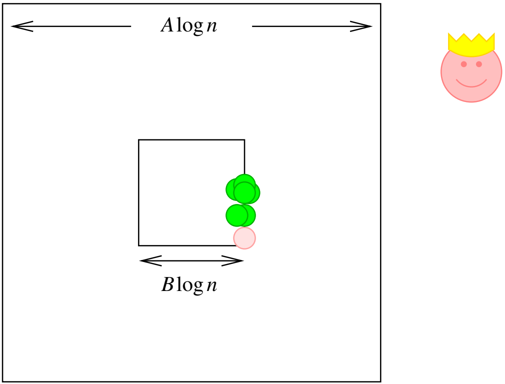
Torus: survivor's strategy



Torus: survivor's strategy



Torus: survivor's strategy



Open questions

- Upper bound on $z(T_n)$.
- Mixed cop-zombie model: how many cops are needed to lead a team of zombies?

Thank you