Calculus II Project 3: Periodic Population Fluctuations

DUE DATE: SOME DATE

Guidelines: This project is a group project which is based on material found in Section 9.5 (pages 458-468), Fourier Series. Since this section will not be covered in lecture, you should first read this material before attempting the problems. Remember that part of your grade will be based on the quality of your written work. The paper you turn in should be a mix of equations, formulas and prose. Graphs may be copied from your calculator, but should be clearly labeled. Use complete sentences, good grammar, correct spelling and correct punctuation. You should write your answers in such a way that your report can be read and understood by anyone who knows the material for this course. This is your target audience, not the instructor. Finally, neatness counts, so the project should be neatly typed or written on good paper (not torn from a notebook).

About Group Projects. To get everyone involved and the group functioning smoothly, it is a good idea to meet as early as possible to arrange meeting times, etc. It might be helpful to bear in mind that there are at least four roles to be played by various participants at various times: the chair, reporter, scheduler and scribe. The role of the chair is to try to get everyone involved and make sure everyone understands the ideas developed by the group. The reporter jots down the ideas of the group as they are discussed. The scheduler finds times and places where everyone in the group can meet, and finally, the scribe writes up the final report for the group. These jobs can be rotated on a per meeting basis if the group wishes. However, everyone must proofread the final draft.

When the project is turned in, students may be asked to evaluate the level of participation by other group members by way of a project participation report to be filled out by each member individually and turned in to the recitation instructor.

This project comes in the form of a memo from a division manager.

North American Ecology Corporation Standard Memo Form

Date: 10/18/01
To: Math analysis team
From: J. Datapoint, Manager, data collection team
Subject: Melanaplus Bivittatus Life Cycle Project

As you know, our data collection team was assigned the task of tracking the life cycle of the *Melanaplus bivittatus* (two-striped grasshopper in common parlance). We found an uncultivated location in Texas where the population was fairly stable from year to year, and that is where we collected population data over a period of three years. The ultimate goal of the Life Cycles Project is to find ecologically sound ways to control grasshopper populations and minimize the use of pesticides. In order to do so, we need to understand how their population changes and what factors cause these changes. Possible factors include the weather, predators, parasites, infections and, of course, food supply. Interestingly, most of these factors tend to be seasonal, and will spike periodically during the year, possibly more than once. If you're curious about the biology, you might check out web sites such as `http://www.sdvc.uwyo.edu/grasshopper`, but what we want from you is some mathematical analysis. Our biologists will take it from there.

The Model. We know that the population $P(t)$ as a function of time is approximately periodic. We are passing on to you the average monthly population count (in thousands per acre), starting with January and ending with December. These counts were taken on twelve equally spaced days over the year, roughly the first day of each month. Our statisticians assure us that these figures will likely have no more than 3% error. We want to use Fourier analysis to resolve the population function into harmonics and determine the energy of each harmonic. This will give us clues as to what kind of periodic factors to look for and which are the most important. We think that the actual population function $P(t)$ should only have a few harmonics. In fact, there will be none beyond than the first six harmonics since we are sure that there are no periodic factors with a period of less than two months.

The Problem. Here is the information we need from you: we want to know what you think are the significant harmonics. Remember that the data has as much as 3% noise, so relatively small harmonics are probably just noise and should be discarded, that is, counted as zero. We would like some explanation as to how you selected the significant harmonics. We would also like to see a plot of the resulting energy spectrum and phase spectrum of $P(t)$. We want to know the first day in the year that each harmonic peaks (reaches a maximum). We also want to see a plot of the data points and the Fourier series approximation for $P(t)$ that results from your analysis using the significant harmonics. Finally, discuss your results as you present them.

Some Suggestions. Here are a few suggestions we have for you.

(a) We remind you that if the kth harmonic of $P(t)$ is $a_k \cos kt + b_k \sin kt$, then its *amplitude* is $A_k = \sqrt{a_k^2 + b_k^2}$ and its *energy* is A_k^2. The *phase* of the harmonic is the angle ϕ_k such that $-\pi \le \phi_k \le \pi$ and $a_k \cos kt + b_k \sin kt = A_k \sin(kt + \phi_k)$. Thus, $\sin \phi_k = a_k / A_k$ and $\cos \phi_k = b_k / A_k$. The population function's *energy and phase spectra* are the plots of A_k^2 and ϕ_k, respectively, against k. Once you know the phase angle, it's easy to find the peaks of the k-th harmonic. The energy of the 0th harmonic is defined differently as A_0^2, where $A_0 = \sqrt{2} a_0$.

(b) To keep things simple, do your calculations on the interval $[0, 2\pi]$ so that 2π time units represent one year. But answer our peaking questions in terms of days, day 1 being January 1.

(b) You will need to approximate the Fourier coefficients using the integral definition and some numerical integration method. In the case of a periodic smooth function, the simple left Riemann rule is highly effective and equivalent to the trapezoidal method. Of course, answers are only approximate, so there are two sources of error: the error of our measurements and the error of numerical integration. Since there are only 12 data points, LEFT(12) is the best choice.

(c) Plotting will help. Certainly, if you omit a significant harmonic, you should get a poor approximation to the data. Also, the plot of your approximation to $P(t)$ should give a reasonably good approximation to the data as should the Fourier series of the data (including insignificant harmonics). TI-8x users can use 2nd STAT PLOT for displaying discrete data plots.

Data. Here are the experimentally observed populations for twelve months, beginning with January 1 and ending with December 1:

0.137, 0.146, 0.220, 0.703, 1.435, 1.508, 1.857, 2.879, 3.154, 3.058, 2.670, 1.192