Calculus II Project 2: Probability and Distributions

DUE DATE: SOME DATE

Guidelines: This project is a group project which is based on material found in the section on probability and more distribution functions in the text (pages 416-423). Since this section will not be covered in lecture, you should first read this material before attempting the problems. Remember that part of your grade will be based on the quality of your written work. The paper you turn in should be a mix of equations, formulas and prose. Graphs may be copied from your calculator, but should be clearly labelled. Use complete sentences, good grammar, correct spelling and correct punctuation. You should write your answers in such a way that it can be read and understood by anyone who knows the material for this course. Finally, neatness counts, so the project should be neatly typed or written on good paper (not torn from a notebook).

About Group Projects. To get everyone involved and the group functioning smoothly, it is a good idea to meet as early as possible to arrange meeting times, etc. It might be helpful to bear in mind the there are at least four roles to played by various participants at various times: the chair, reporter, scheduler and scribe. The role of the chair is to try to get everyone involved and make sure everyone is understanding the ideas developed by the group. The reporter jots down the ideas of the group as they are discussed. The scheduler finds times and places where everyone in the group can meet, and finally, the scribe writes up the final report for the group. These jobs can be rotated on a per meeting basis if the group wishes. However, everyone should proofread the final draft and help in the other duties as they see fit.

When the project is turned in, students will be asked to evaluate the level of participation by other group members by way of a project participation report to be filled out by each member individually and turned in to the recitation instructor.

This project comes in the form of a memo from a division manager.

Intelligent Communications Corporation Standard Memo Form

Date: 10/31/00
To: Math analysis team
From: J. Datapoint, Manager, statistics analysis team
Subject: Reliability of our Talk Now line

As you know, our Talk Now two-way radio systems have been quite a success. Recently, however, we have had a large number of warranty returns within the one year warranty period. These returns seems to be localized at a small number of retail sites, and the question we need to answer is whether or not these warranty returns are reasonable. If not, we'll need to stop sales at these sites or send a team to track down the difficulties, which could be expensive. So management asked us to review our model, which we formulated during the development phase of the Talk Now project, and send information about the our model to you. Your role in this is to use calculus techniques (and only calculus techniques) to confirm our conclusions or draw new conclusions about this situation (we aren't going to tell you what ours were, so as not to bias your analysis). We would like you to write a report on this subject, intelligible to anyone with about two semesters of calculus under her/his belt.

The Model. We know that the density function for the lifetime x of a Talk Now system is given by

$$p(x) = Ce^{-x/b}$$

where b is a positive number, called the *reliability factor* of the density function, and C is a positive constant that is defined by the requirement that $p(x)$ be a density function. Of course, the value of C depends on the reliability factor b. This type of density function is fairly typical for electronic devices like the Talk Now system. What about reliability factors? We know that this varies with retail sites, and depends on other conditions, like the factory that manufactured it, general climate at the site, quality of the installation, etc. To the best of our knowledge, the reliability factor b for systems at a given site is a property of that site such that the density function $q(b)$ for b is a normal distribution with mean $\mu = 2$ and standard deviation $\sigma = 0.4$. For all practical purposes $q(b)$ is zero outside the interval $[\frac{1}{2}, \frac{7}{2}]$ (we've never seen reliability factors outside this range).

The Problem. Here is the information we need from you: (1) management wants to know the reliability factor b_0 for which the probability of a smaller reliability factor occurring is at most .05. (2) Most importantly, with this reliability factor, what is the probability that a TNS will fail within the first year? Management wants to use this information to decide what retail sites are doing an unacceptable job. (3) Management recently found a site where the warranty returns were at 60%. They think that this is so unlikely that fraud or gross incompetence is suspected. What do you think? (4) Finally, our department would like you to compute the mean value for the lifetime of a TNS using this reliability factor. We'd like a few nice sketches of the density functions and cumulative distribution functions in question as well.

Some Suggestions. Here are a few suggestions we have for you.

(a) You will need the value of the constant C of the density function $p(x)$ and the cumulative distribution function $P(x)$ corresponding to $p(x)$. You should be able to compute these exactly and determine a formula for the constant C in terms of the reliability factor b.

(b) You should also be able to compute the mean value of the lifetime and the probability of failure within one year exactly.

(c) You probably won't be able to compute the cumulative distribution function $Q(b)$ corresponding to the density function $q(b)$, but you can use your calculator and numerical techniques to approximate $Q(b)$ for any particular b. Trial and error will help you find (approximately) the reliability factor b_0 that we're looking for.

(d) Once you've found this reliability factor, plot the resulting density function $p(x)$ and cumulative distribution function $P(x)$. Also plot the density function $q(b)$ and (approximately) the cumulative distribution function $Q(b)$.