

Calculus II Project 1: Density and Distribution Functions

August 18, 2018

DUE DATE: SOME DATE

Guidelines: This project consists of three independent problems. These problems concern the material found in the section on distribution functions section in the text (pages 409-415). Since this section will not be covered in lecture, you should first read this material before attempting the problems. Remember that part of your grade will be based on the quality of your written work. The paper you turn in should be a mix of equations, formulas and prose. Graphs may be copied from your calculator, but should be clearly labelled. Use complete sentences, good grammar, correct spelling and correct punctuation. You should write your answers in such a way that it can be read and understood by anyone who knows the material for this course. Also, your document should be reasonably self-contained.

Problem 1. For each of the following descriptions of populations, sketch the graph of a density function and a cumulative distribution function which could reasonably represent the distribution of income (x) in that population.

- (a) A large middle class.
- (b) Few rich people but large middle and lower classes.

In each graph of a density function draw a vertical line in such a way that about half the total area between the curve and x -axis is to the left of the vertical line.

Problem 2. A study of grade point averages (GPAs) for students at a certain

university yields the following data:

GPA (G)	Fraction of students with $\text{GPA} \leq G$
1.0	.01
1.5	.04
2.0	.10
2.5	.22
3.0	.40
3.25	.57
3.5	.75
3.75	.89
3.9	.95

In sketching the curves described below, use the same scale for the horizontal (G) axis in both cases.

(a) Plot the data above with G on the horizontal axis. Sketch the best curve you can through these points. Is this a density function or a cumulative distribution function?

(b) If you think you drew a density function in part (a), sketch the corresponding cumulative distribution function. If you think you drew a cumulative distribution function, sketch the corresponding density function.

Problem 3. Suppose we are interested in studying the life of an electronic part. Let x denote the lifetime (in years) of the part and suppose that the density function of x has the form (for some constant c)

$$p(x) = \begin{cases} cxe^{-x^2}, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

(a) Use the definition of density function to show that c must equal 2.

(b) Sketch the graph of $p(x)$.

(c) Find the cumulative distribution function of $p(x)$ and sketch its graph.