Lecture 7 for Math 398 Section 952: Linear Algebra

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Most of the functions that we study in this lesson can be found in one category of the help files. Start by issuing a "helpwin" command and clicking on "matlab/funfun".

Linear Algebra

This is what Matlab was created for! So it should come as no surprise that Matlab has many sophisticated functions for dealing with problems of linear algebra.

Matrix constructors and tools.

Matlab makes it easy to construct structured matrices. Get help and see. Here is a sampling. See if you can guess the general pattern, then get help.

```
> c = (0:5)
> r =0:2:10
> T = toeplitz(c,r)
> fliplr(T)
> triu(T)
> tril(T)
> 5*eye(5)
> T + 5*eye(5)
vander(c)
```

Linear equations.

We will solve an interpolation problem. Suppose we are given a set of points $x_1 < x_2 < ... < x_n$ and we want to find the coefficients of the polynomial of degree $n-1$ that assumes values $y_1, y_2, ..., y_n$ at these points respectively. The vander command is perfect for this. So let's construct a polynomial approximating $\sin(2\pi x)$, $0 \leq x \leq 1$. Here's a simple routine (disclaimer: Matlab has built-in functions to do this job; we're just exploring linear algebra routines.)

```
> n = 5;
> x = linspace(0, 1, n)';
> V = vander(x)
> y = sin(2*pi*x)
> p = V\y
```

```
> xnodes = linspace(0, 1, n^2);
> plot(xnodes,sin(2*pi*xnodes));
> hold on, grid
plot(xnodes, polyval(p,xnodes),'r');
```

This seems nice. So let's try to rev things up. Repeat all of the above after closing the window and setting n = 100.

Eigenvalues and eigenvectors.

We will do a simple eigenvalue problem.

```
> [M,D] = eig(T)
> inv(M)*T*M
```

Now for a hard one. Get out your stopwatches:

```
> [M, D] = eig(V);
```

SVD and condition numbers.

Back to our problem with interpolation. The condition number is a matrix gives a measure of it's sensitivity to change, and hence how badly it might distort the solution, when we try to solve a system with the matrix as a coefficient. Larger condition numbers are bad, and the smallest it can be is 1. Try this

```
> cond(V)
> cond(T)
[U, S, V] = svd(T)
U'*T*V
S(1,1)/S(6,6)
cond(T)
```

End of Lesson