Math 4/896: Seminar in Mathematics
Topic: Inverse Theory

Instructor: Thomas Shores
Department of Mathematics

AvH 10

Some references:

1. C. Groetsch, *Inverse Problems in the Mathematical Sciences*, Vieweg-Verlag, Braunschweig, Wiesbaden, 1993. (A charmer!)
2. M. Hanke and O. Scherzer, *Inverse Problems Light: Numerical Differentiation*, Amer. Math. Monthly, Vol 108 (2001), 512-521. (Entertaining and gets to the heart of the matter quickly)
3. A. Kirsch, *An Introduction to Mathematical Theory of Inverse Problems*, Springer-Verlag, New York, 1996. (Harder! Definitely a graduate level text)
4. A. Tarantola, *Inverse Problem Theory and Methods for Model Parameter Estimation*, SIAM, Philadelphia, 2004. (Very substantial introduction to inverse theory at the graduate level that emphasises statistical concepts.)
5. R. Aster, B. Borchers, C. Thurber, *Estimation and Inverse Problems,* Elsivier, New York, 2005. (And the winner is...)

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Outline

Linear Algebra and Matlab

For this portion of the lecture, open copies of the two files LinearAlgebraLecture-496s6.pdf and MatlabLecture-496s6.pdf. We will work our way through the Matlab tutorial, especially the linear algebra parts, with occasional reference to the linear algebra notes.

Probability and Statistics Preliminaries

1. Let's start with questions on Matlab and the Matlab tutorial, with a brief overview of the part of the tutorial we did not cover.
2. Save a copy of `Math496s6Lecture3.pdf` and `ProbStatLecture-384H.pdf` to your local drives.
3. Save copies of all m files in Week2 to your local drive.
4. Alternative to 2-3: save a copy of Week2.zip and unzip it.
5. Open up the pdf files and fire up Matlab. Get help on `addpath` and use it to put your m files in Matlab's path.

Credit and Thanks: The m files we're using today are found on the outstanding site of James P. LeSage at `http://www.spatial-econometrics.com/`

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Outline

Discrete Example

After viewing the probability discussion in ProbStatLecture:

Dart board experiment:

The dart board consists of six regions of equal area, and dart is thrown without bias to any region.

Answer these questions:

1. What is the probability of landing in any one region?
2. Suppose the experiment is repeated once. What is the probability of the event of both darts landing in the same region?

Simulating an Experiment

Let's simulate the dart experiment and graph the results of our experiments using Matlab. Type in

```
> N=36
> x = rand(N,1)*6;
> hist(x,0.5:5.5)
```

Now repeat with larger N.

Outline

Uniform Distribution

After viewing the statistics discussion through expectation and variance in ProbStatLecture:

Uniform distribution:

Let's take the case of $[a, b] = [0, 1]$.

Answer these questions:

1. What does the graph of the p.d.f. look like?
2. How does simple calculus help us find the c.d.f.?
3. Can we calulate the expectation and variance of the distribution?
4. Can we verify a simple property of expectation and variance from definition?

Normality and Central Limit Theorem

Normal distributions:

Let's focus on the standard normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < \infty$$

After viewing discussion of normal distributions and the Central Limit Theorem, use addpath to point to the distribution files. Then

```
> x = -10:.1:10;
> help norm_cdf
> y = norm_cdf(x,0,1);
> plot(x,y)
> hold on
> help norm_pdf
> y = norm_pdf(x,0,1);
> plot(x,y)
```

Outline

Common Distributions

After viewing the discussion of common distributions in ProbStatLecture:

1. Do a simple plot of the normal distributions $N(0, \sigma)$, $\sigma = 0.5, 1.0, 2.0, 3.0$.
2. Confirm the approximation assertion about Poisson vs binomial by calculating certain values or plotting.
3. Get an idea of the shapes of non-normal distributions as one of their parameters vary.
4. Confirm the limiting assertion about the Student's t distribution.
5. Design an experiment to confirm the application of Student's t.