

Math 428: Introduction to Operations Research

Instructor: Thomas Shores
Department of Mathematics

Week 1, January 15, 2008
AvH 12

Outline

- 1 Chapter 1: Introduction
- 2 Chapter 2: Overview of OR
- 3 Chapter 3: Introduction to Linear Programming

Definitions

Operations Research:

What does it mean?

- “Research on operations, so applied to problems concerning how to conduct and coordinate operations in an organization.”
- “The application of mathematical methods, e.g., optimization, to study or plan a process.”
- “Management science” ... perhaps in the sense of da Vinci: “No human investigation can be called real science if it cannot be demonstrated mathematically.”
- None is quite right. What about statistical or computer science techniques, both of which are found in this text? What about human management issues such as extracting the real problem from less quantitatively oriented colleagues?

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Origins

Origins:

- War needs, especially WWII, with processes like logistics planning, resource allocation, etc.
- Dantzig (~1947) was first to formalize the simplex method for linear programming problems, arguably the most fundamental tool of OR.
- Kantoravich (~1940) derived methods for constrained systems that predated Dantzig.
- Hillier and Lieberman's classic text (1967-present) popularized the discipline for students of management science.

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HUGE:

- HP, 1998: Redesign sizes and locations of buffers in printer production line to meet production goals. Increase in revenue ~\$280M
- China, 1995: Optimally select and schedule massive projects for meeting country's future energy needs. Savings of ~\$425M
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On the text CD:

- Student versions of LINGO/LINDO – commercial modeling language and engine.
- MPL/CPLEX – another commercial modeling language and engine.
- OR tutoring programs and examples – and if we can get the *@_~& program to work reliably, we'll use it now and then.
- **Excel enhancements** – such as Solver – these are the principal tools that we will use in this course.

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A Modeling Approach

Summary:

- 1. Define the problem of interest and gather relevant data. (Users may not even understand their problem – you have to clarify it.)
- 2. Formulate a mathematical model. (Basically, choose decision variables x_1, x_2, \dots, x_n , constraints on decision variables and objective functions of the decision variables to be optimized.)
- 3. Derive solution(s) from the model and do post-optimality analysis such as sensitivity of solutions, “what-if’s”, etc.
- 4. Test the model. (Do answers make sense, can you test against prior data, are there blunders, mathematical or numerical instabilities?) If tests fail, goto 1.

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A Modeling Approach

Summary (continued):

- 5. Prepare to apply the model. (Develop a full, reliable, usable system for end users.)
- 6. Implementation. (Implement as prescribed by management.)
- In this course, we will primarily focus of items 1–4.

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What It Is Not

First a disclaimer: Here “programming” does not mean “computer programming” but rather, “planning”. So these models are called “linear programs.”

Much of the work we do will be by hand. This would never be done in the real world, but our objectives include understanding the methods at a higher level, and you have to get your hands dirty to do that.

Nonetheless, the algorithms that come from linear programming are very programmable, and we will use software that does the grunt work for more detailed and realistic problems.

The Wyndor Glass Co.

The problem:

Wyndor produces high quality glass products, including windows and doors. Plant 1 produces aluminum frames and hardware, Plant 2 produces wood frames and Plant 3 produces glass and assembly.

- Objective is to maximize profit.
- An opportunity has arisen to produce two new products of large sales potential:
 - Product 1: An 8-foot glass door with aluminum framing.
 - Product 2: a 4x6 foot double-hung wood-framed door.
- Relevant variables are
 - weekly production level of each product (produced in batches of 20, so level = # batches per week).
 - Profit resulting from weekly production levels

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Linear Programming Formulation

Grab and open a copy of the WyndorGlass.xls file in Week 1.

The problem:

Express the Wyndor model described above mathematically and solve it graphically.

- Decision variables: $x_1 = \#$ of batches of Product 1 per week and $x_2 = \#$ of batches of Product 2 per week.
- Objective function: $Z = 3x_1 + 5x_2$, where Z is measured in thousands of dollars.
- Constraints:
 - Plant 1 constraint $x_1 \leq 4$.
 - Plant 1 constraint $2x_2 \leq 12$.
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 - Variables constraints: $x_1 \geq 0, x_2 \geq 0$.
- Now express this problem in standard matrix formulation!

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