Exam 3 **Math 314** **Sample**

Name:______________________________ **Score:**____________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

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(27) **1.** Let $\mathbf{v}_1 = (-1,-1,1,1)$, $\mathbf{v}_2 = (1,1,1,1)$, and $\mathbf{v}_3 = (-1,1,-1,1)$.

(a) Show that $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ is an orthogonal set of vectors and convert it to an orthonormal set $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ by normalizing each vector.

(b) Use the orthogonal coordinates theorem to determine if $(4,5,0,1)$ belongs to span $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.

(c) Set up and solve the normal equations for the system $A\mathbf{x} = \mathbf{b}$, where$A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$ and $\mathbf{b} = (-1,0,0,1)$.

(15) **2.** Let $A = \begin{bmatrix} -4 & 6 \\ -3 & 5 \end{bmatrix}$. Then the eigenvalues of A are $-1, 2$ and two eigenvectors are $(1,1)$ and $(2,1)$. Assume this information and use it to derive a formula for the powers of A in terms of the eigenvalues of A.

(20) **3.** (a) Find all eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(b) Is the matrix A diagonalizable? If not, give reasons, otherwise compute the matrix P that diagonalizes A.

(18) **4.** Fill in the blanks or answer T/F:

(a) Every real matrix is diagonalizable (T/F) ____________.

(b) Every orthonormal set of vectors is linearly independent (T/F) ____________.

(c) Eigenvalues of a matrix cannot be zero (T/F) ____________.

(d) The CBS inequality says that for vectors $\mathbf{u}$ and $\mathbf{v}$, we have that ________________________.

(e) Every eigenvalue of a Hermitian matrix is real (T/F) ____________.

(f) The projection of $\mathbf{u} = (1,2,0,1)$ along the vector $\mathbf{v} = (1,1,1,0)$ is $\text{proj}_{\mathbf{v}}(\mathbf{u}) =$________________ and the orthogonal projection of $\mathbf{u}$ to $\mathbf{v}$ is $\text{orth}_{\mathbf{v}}(\mathbf{u}) =$________________.

(g) The component of $\mathbf{u} = (-3,2,1,0)$ along the vector $\mathbf{v} = (0,2,-1,1)$ is $\text{comp}_{\mathbf{v}}(\mathbf{u}) =$________________ and the cosine of an angle between $\mathbf{u}$ and $\mathbf{v}$ is ________________.

(20) **5.** Give brief answers to the following questions.

(a) The matrix $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ has eigenvectors $(i,1)$ and $(1,i)$. Assume this and unitarily diagonalize A, that is, produce the appropriate matrices U and D such that $U^*AU = D$.

(b) Suppose that the diagonalizable matrix A is the transition matrix of a discrete dynamical system and that the spectral radius (largest eigenvalue in absolute value) of A is 1. What can you say about the states $\mathbf{x}^{(k)} = A^k \mathbf{x}^{(0)}$?

(c) Define what an eigenvector and eigenvalue of the matrix A is. Use this to show that if λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.