Exam 2 **Math 314** **Sample**

Name:______________________________ **Score:**____________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

(25) **1.** In the following you are given that

$$A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5] = \begin{bmatrix} 1 & -1 & 5 & 1 & 0 \\ 2 & 1 & 4 & 2 & 1 \\ 3 & 0 & 9 & 3 & 1 \\ -1 & -1 & -1 & -1 & 2 \end{bmatrix} \text{ has reduced row echelon form } E = \begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) What is the rank of A and dimensions of $\mathcal{R}(A)$, $\mathcal{C}(A)$ and $\mathcal{N}(A)$?

(b) Find a basis for the row space of A.

(c) Find a basis for the column space of A.

(d) Find a basis for the null space of A.

(e) Express $\mathbf{v}_3$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4, \mathbf{v}_5$.

(12) **2.** In the following, $\mathbf{u}_1 = (1, 0, 1)$ and $\mathbf{u}_2 = (1, -1, 1)$.

(a) Show that $\mathbf{u}_1, \mathbf{u}_2$ form a linearly independent set.

(b) Does $\mathbf{v} = (2, 1, 2)$ belong to span$\{\mathbf{u}_1, \mathbf{u}_2\}$? Give reasons for your answer.

(c) Find a basis of $\mathbb{R}^3$ which contains $\mathbf{u}_1$ and $\mathbf{u}_2$. Give reasons for your answer.

(12) **3.** Use the subspace test to decide if W is a subspace of the given (abstract) vector space V.

(a) V is the space of all 2×2 matrices over the reals with the usual matrix addition and scalar multiplication and W is the set of 2×2 matrices of the form $\left\{ \begin{bmatrix} a & b \\ 0 & b \end{bmatrix} \mid a, b \text{ are reals} \right\}$.

(b) V is the space $C[0, 1]$ of continuuous functions on [0,1] and W is set of f in $C[0, 1]$ such that $f(0) = 2$.

(12) **4.** Let $T : \mathcal{P}_2 \to \mathbb{R}^2$ be the linear operator defined by

$$T\left(a + bx + cx^2\right) = \begin{bmatrix} a + c \\ a - b \end{bmatrix}.$$

(a) Find the kernel and range of T.

(b) Is this operator one-to-one? Onto? An isomorphism? Explain.

(24) **5.** Fill in the blanks or answer T/F:

(a) Every spanning set of a vector space contains a basis of the space (T/F) ____________.

(b) Every vector space is finite dimensional (T/F) ____________.

(c) The vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ are linearly dependent means that

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(d) Every subspace of a finite dimensional space is itself finite dimensional (T/F) ____________.

(e) The span of the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in a vector space is the set of all vectors $\mathbf{v}$ of the form $\mathbf{v} =$

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(f) The set consisting of the zero vector is a linearly independent set (T/F) ____________.

(g) The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (x + y, x - 2xy)$ is linear (T/F) ____________.

(h) The adjoint of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ is ____________

(i) The cofactor matrix of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A_{\text{cof}} =$ ____________.

(j) The inverse of a matrix A with nonzero determinant is $\frac{1}{\det A} A_{\text{cof}}$ (T/F) ____________.

(k) The dimension of $\mathbb{C}$ as a vector space over $\mathbb{R}$ is ____________.

(l) The Dimension Theorem asserts that any two bases of a vector space ____________.

(15) **6.** Suppose that the linear system $A\mathbf{x} = \mathbf{b}$ is a consistent system of equations, where A is an mxn matrix and $\mathbf{x} = [x_1, ..., x_n]^T$. Prove the following:

(a) $\mathbf{b} \in \mathcal{C}(A)$.

(b) If $\mathbf{x}_0$ is any particular solution to the system, then every vector of the form $\mathbf{x}_0 + \mathbf{z}$, where $z \in \mathcal{N}(A)$, is a solution to the system.

(c) If the set of columns of A has redundant vectors in it, show the system has more than one solution.