Exam 1 **Math 314** **Sample**

Name:_____ **Score:**_____

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

(25) **1.** In each of the following problems, clearly identify the augmented matrix of the system and give its rank.
(i) Use Gauss-Jordan elimination to find the general solution to this system:

$$\begin{aligned} x_1 + x_2 + x_4 &= 1 \\ 2x_1 + 2x_2 + x_3 + x_4 &= 1 \\ 2x_1 + 2x_2 + 2x_4 &= 2 \end{aligned}$$

(ii) Use Gaussian elimination with back solving to find the solution to this system. Here, b_1, b_2 are constants and x_1, x_2 the unknowns.

$$\begin{aligned} x_1 - x_2 &= b_1 \\ 2x_1 + 2x_2 &= b_2 \end{aligned}$$

(14) **2.** Given the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ and vector $\mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$, find the inverse of the matrix A and use this to solve the system $A\mathbf{x} = \mathbf{b}$.

(13) **3.** Calculate the following:
(i) The solutions to the equation $z^3 = -8$. Express them in polar form and graph them in the complex plane.

(ii) $\dfrac{2+i}{2-i} =$

(iii) $\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 0 \end{vmatrix} =$

(20) **4.** Carry out these calculations or indicate they are impossible. You are given that $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 3 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 1+i & 2 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$.
(a) ba

(b) $\mathbf{ab}$

(c) $C\mathbf{b}$

(d) $\mathbf{b}D$

(e) $D + D^T$

(f) C^*C

(g) C^{-1}

(18) **5.** Fill in the blanks or answer True/False (T/F):
(a) If P and Q are invertible and A a matrix such that PAQ is defined, then rankPAQ =_____

(b) If A is a 3×3 matrix, then in terms of $\det(A)$, we can say that $\det(-2A) =$ _____.

(c) As an elementary matrix, $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is _____.

(d) As elementary matrices, $E_{ij}(a)^{-1} =$ _____ and $E_{ij}(a)^T =$ _____.

(e) If two rows of a determinant $|A|$ are interchanged then the new determinant is _____.

(f) If $A\mathbf{x} = \mathbf{b}$ has a unique solution for some particular $\mathbf{b}$ then A is invertible (T/F):

(g) Every nonzero square matrix has an inverse (T/F):

(h) Any homogeneous linear system with more unknowns than equations has a nontrivial solution (T/F):

(i) The rank of the matrix A is the same as the rank of A^T (T/F):

(j) For 2×2 matrices A and B, $AB = BA$ (T/F):

(k) If A and B are matrices such that $AB = 0$, then $A = 0$ or $B = 0$ (T/F):

(l) If A is both upper triangular and lower triangular, then A is diagonal (T/F):

(10) **6.** Give the definition of an invertible matrix. Then use the definition and basic facts to prove that if A is invertible and A is symmetric, then so is A^{-1}.