Quiz 1 **Math 221H** **Fall 2008**

Name:______________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than $\sin \pi$. Point values of problems are given in parentheses. Notes or text in *any* form are not allowed. The only electronic equipment allowed is a calculator.

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(6) **1.** (Exer. 1.2.15) Find the position function $x(t)$ of a moving particle with acceleration $a(t) = 4(t+3)^2$, initial velocity $v_0 = -1$, and initial position $x_0 = 1$.

SOLUTION. Let $x(t)$ be the position at time t with $x(0) = x_0 = 1$, and $v(t) = x'(t)$ the velocity at time t with $v(0) = v_0 = -1$. The acceleration is given as

$$a(t) = v'(t) = x''(t) = 4(t+3)^2$$

so that $v(t) = \int a(t)\,dt = \frac{4}{3}(t+3)^3 + C$. Thus $v(0) = -1 = \frac{4}{3}3^3 + C$ and $C = -1 - 36 = -37$. Next,

$$x(t) = \int v(t)\,dt = \int \left(\frac{4}{3}(t+3)^3 - 37\right) dt = \frac{1}{3}(t+3)^4 - 37t + D$$

. At $t = 0$ we obtain that $x(0) = 1 = 3^3 + D$, so that $D = -26$ and thus

$$x(t) = \frac{1}{3}(t+3)^4 - 37t - 26 = \frac{1}{3}t^4 + 4t^3 + 18t^2 - t + 1.$$

(8) **2.** (Exer. 1.3.19) Determine whether or not the existence and/or uniqueness of Theorem 1 (Key Fact in class) applies to the IVP $\frac{dy}{dx} = \ln\left(1 + y^2\right)$, $y(0) = 0$. (give reasons!)

SOLUTION. The right hand side is $f(x,y) = \ln(1+y^2)$, which is continuous for all x and y. So the point $(0,0)$ certainly fits inside a box R (any box!) in which $f(x,y)$ is continuous. Hence, the existence of a solution in some interval containing 0 on the x-axis is guaranteed by the Key Fact. Similarly, $f_y(x,y) = \frac{2y}{1+y^2}$,which is continuous for all x and y, which includes includes any box R with $(0,0)$ in its interior, so the solution described above must also be unique by the Key Fact.

(6) **3.** (Exer. 1.4.12) Find general solutions to the DE $y\,y' = xy^2 + x$.

SOLUTION. We have that $y\frac{dy}{dx} = x(y^2+1)$. Divide this equation by $y^2 + 1$ and multiply by dx to obtain

$$\frac{y\,dy}{y^2+1} = x dx.$$

Integrate both sides to obtain (using substitution $u = y^2 + 1$, $du = 2y\,dy$ on the left) to obtain

$$\ln\left(y^2+1\right) = 2\frac{x^2}{2} + C,$$

where C is a constant of integration. Take exponentials and obtain the implicit equation

$$y^2 + 1 = e^C e^{x^2} = Ae^{x^2}, \text{ or if you solve explicitly, } y = \pm\sqrt{Ae^{x^2} - 1},$$

where $A = e^C$ is a positive constant. [Note: I didn't take any points off for this, but in fact, you should say that $A \geq 1$, because evaluating at $x = 0$, gives $Ae^0 = A = y(0)^2 + 1 \geq 1$.]