Sample Final **Math 221H** **Fall 2008**

Name:____________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Clearly identify answers and show supporting work to receive any credit. Give exact answers (e.g., π) rather than inexact (e.g., 3.14); make obvious simplifications, e.g., 0 rather than $\sin\pi$. Point values are in parentheses. Notes, text or electronic equipment not allowed. Table entries for Laplace transforms may be used freely, unless otherwise stated. However, you must show enough detail so that it is clear that you are using table entries.

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(12) **1**. Classify each of the following systems as linear(L) or nonlinear (NL), autonomous(A) or non-autonomous (NA), and separable (S) or non-separable (NS).
(a) $xy' = y + e^y$
(b) $y\dfrac{dy}{dx} = xy - 1$
(c) $xy' = xy + 1$
(d) $\frac{dx}{dt} = x + yx$, $\frac{dy}{dt} = -x + y$

(14) **2**. (a) Solve the IVP $\frac{dy}{dx} + 2xy = 0$, $y(0) = 1$.
(b) Solve the DE $y' - 2xy = e^{x^2}$.

(12) **3**. Use Euler's Method with a stepsize of $h = 1/3$ to the IVP $\frac{dy}{dx} = xy$, $y(0) = 1$ on the interval $[0, 1]$ to obtain an approximation to $y(1)$.

(40) **4**. (a) Find two solutions to the homogeneous problem $x'' - 2x' + 2x = 0$.
(b) Use (a) and the method of undetermined coefficients to find the general solution to the DE $x'' - 2x' + 2x = t + 1$.
(c) Find the solution to the IVP given by the differential equation in (b) and initial conditions $x(0) = 0$, $x'(0) = 0$.
(d) Convert the homogeneous system of (a) to a first order linear system, graph a typical solution in the phase plane, and classify the equilibrium point $(0, 0)$. (Hint: sketch a simple solution.)

(16) **5**. Sketch the phase line for the differential equation $y' = y^2(y - 2)$ along a vertical y-axis, classify the equilibrium solutions and then sketch a few representative solutions in the ty-plane.

(16) **6**. At time $t = 0$ a tank of capacity 400 gallons contains 200 gallons of brine solution which contains 100 ounces of salt. Four gallons of pure water is poured into the tank per minute, the brine is continuously mixed and leaves the tank at the constant rate of 2 gallons per minute. Express the problem of finding $S(t)$, the amount of salt in the tank at time t, as an IVP.

(14) **7**. Given that the linear system $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with constant coefficient matrix A has eigenvalue $-1+2i$, with a corresponding eigenvector $\begin{bmatrix} i \\ 2-i \end{bmatrix}$, find the general solution to the system.

(22) **8**. (a) Use the method of eigenvalues to find a general solution to the linear system $x_1' = x_1 + 2x_2$, $x_2' = 2x_1 + x_2$.
(b) Use the solution in (a) to find the solution to the IVP consistsing of this system together with initial conditions $x_1(0) = -1$, $x_2(0) = 0$.

(12) **9**. Compute the following Laplace transforms. Identify which table entries you use.
(a) $f(t) = (t+1)^2 + e^{2t+3}$, $\mathcal{L}\{f(t)\}(s) =$
(b) $f(t) = u(t-2)t^2 + \cos 4t$, $\mathcal{L}\{f(t)\}(s) =$

(16) **10**. Compute the following inverse Laplace transforms. Identify which table entries you use.
(a) $Y(s) = \dfrac{1}{s^2 + 2s - 3}$, $\mathcal{L}^{-1}\{Y(s)\}(t) =$
(b) $Y(s) = \dfrac{2}{s(s+2)^2}$, $\mathcal{L}^{-1}\{Y(s)\}(t) =$

(18) **11**. Use Laplace transforms to solve the IVP $y'' - 3y' + 2y = 1$, $y(0) = 0$, $y'(0) = 1$.

(8) **12**. Compute the Laplacian of the function $f(t)$ given by

$$f(t) = \begin{cases} t+1, & 1 \le t < 4 \\ 0, & \text{otherwise} \end{cases}$$