Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, but clearly so indicate. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., \( \pi \)) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than \( \sin \pi \). Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes, text or electronic equipment not allowed.

(Note: This sample exam is about 1 question too long.)

(18) 1. Solve the IVP \( \frac{dx}{dt} = -2x_1 + x_2, \frac{dx_2}{dt} = 4x_1 - 2x_2, x_1(0) = 1, x_2(0) = 0 \) by the method of eigenvalues. (Arithmetic check: eigenvalues are 0, -4.)

(18) 2. Find the general solution to \( \frac{dx_1}{dt} = -2x_1 + x_2, \frac{dx_2}{dt} = -1x_1 - 4x_2, \) by the method of eigenvalues. (You may assume that -3, -3 are the eigenvalues of this system and \([1, -1]^T\) is an eigenvector.)

(10) 3. Sketch typical phase planes for a linear system \( dx/dt = Ax + By, dy/dt = Cx + Dy \) which has at the origin a
(a) stable node. \hspace{1cm} (b) saddle. \hspace{1cm} (c) center.

(10) 4. Solve the DE \( \frac{dx}{dt} = 2y, \frac{dy}{dt} = -4x \), sketch a few representative solutions and classify the critical point at the origin.

(12) 5. Find the inverse Laplace transforms.
(a) \( Y(s) = \frac{1}{(s + 9)s^2}, \mathcal{L}^{-1}\{Y\} = \)
(b) \( Y(s) = \frac{8}{s^2 - 2s + 5} + \frac{1}{s^4}, \mathcal{L}^{-1}\{Y\} = \)

(12) 6. Find the Laplace transforms.
(a) \( \mathcal{L}\{\sinh t\} = \)
(b) \( \mathcal{L}\{e^{at}t^2 + t^3 \cos(2t)\} = \)

(12) 7. Express \( f(t) = \int_0^t e^v \sin(t - v)dv \) in terms of convolutions. Then use this expression to find the Laplace transform of \( f(t) \).

(18) 8. Solve the IVP \( y'' - 4y = e^{2t}, y(0) = 0, y'(0) = 0 \) by the method of Laplace transforms.