Sample Exam 1 Math 221H Fall 2008

Name: ________________________ Score: ______

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, but clearly so indicate. You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., \( \pi \)) are preferred to inexact (e.g., 3.14). Make all obvious simplifications, e.g., 0 rather than \( \sin \pi \). Point values of problems are given in parentheses. Point values of problems are given in parentheses. Notes or text in any form not allowed. No electronic equipment is allowed.

(16) 1. Specify the order of the following ODEs and classify (do not solve) the first order ODEs as autonomous(A), exact(E), linear(L) or separable(S), if any. Also indicate what method you would use to solve them (do not actually do it.)

(a) \( yy' = x \left( y^2 + 1 \right) \)
(b) \( yy'' + (y')^2 = 0 \)
(c) \( \frac{dy}{dx} - \frac{y}{2x} = 2xy^{-1} \)
(d) \( \left( 3x^2 + 2y^2 \right) + \left( 4xy + 6y^2 \right) \frac{dy}{dx} = 0 \)

(14) 2. A tank contains 1000 liters (L) of a solution of 100 kg of salt dissolved in water. Pure water is pumped in at a rate of 5 L/s and the stirred solution is pumped out at the same rate. Assume that \( S(0) = 100 \), find a differential equation condition for the amount \( S(t) \) of salt in the tank at time \( t \) and solve for \( S \).

(27) 3. Find all solutions to these DEs or IVPs:

(a) \( \frac{dy}{dx} = y \sin x \)
(b) \( (4x - y) + (6y - x) \frac{dy}{dx} = 0 \)
(c) \( y' + 3y = 2xe^{-3x}, \ y(0) = 2 \)

(12) 4. The acceleration of a Maserati is proportional to the difference between 250 km/hr and the velocity of this sports car. Set up a DE for velocity of this car, sketch its phase diagram and sketch a few representative solutions to the DE.

(16) 5. You are given the DE \( \frac{dy}{dx} = \sqrt{y} \).

(a) Find the equilibrium solutions to this DE and classify them as stable or unstable.
(b) Determine the extent to which the existence/uniqueness theorem applies to the given DE with initial condition \( y(0) = 0 \).

(15) 6. A population is modeled by the autonomous differential equation \( \frac{dx}{dt} = 4x - x^2 - h \) with parameter \( h \). Sketch the bifurcation diagram for this DE and label the critical points as stable or unstable. Sketch some typical solutions to the DE for \( h = 2 \).